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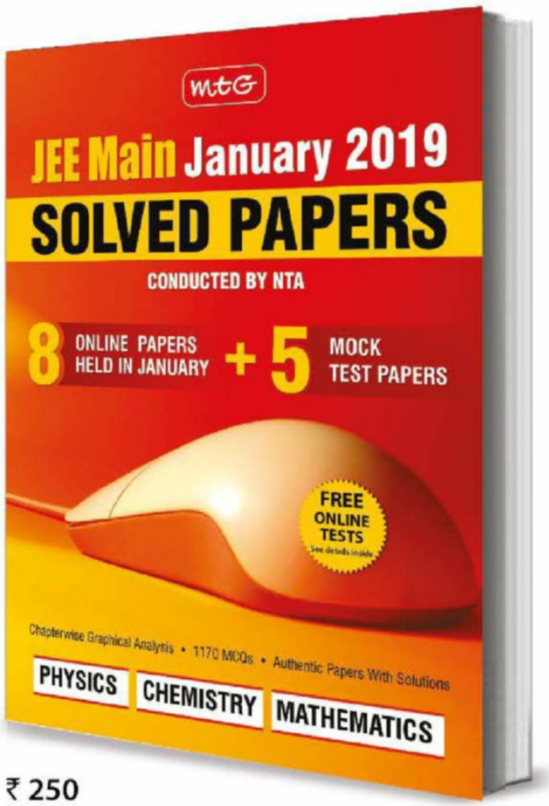

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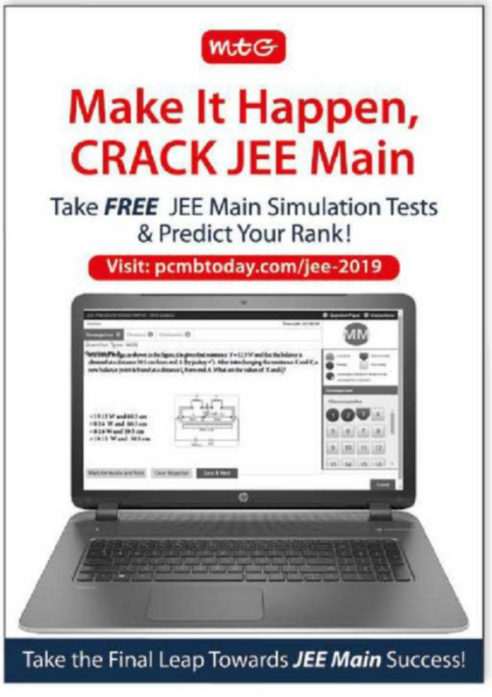

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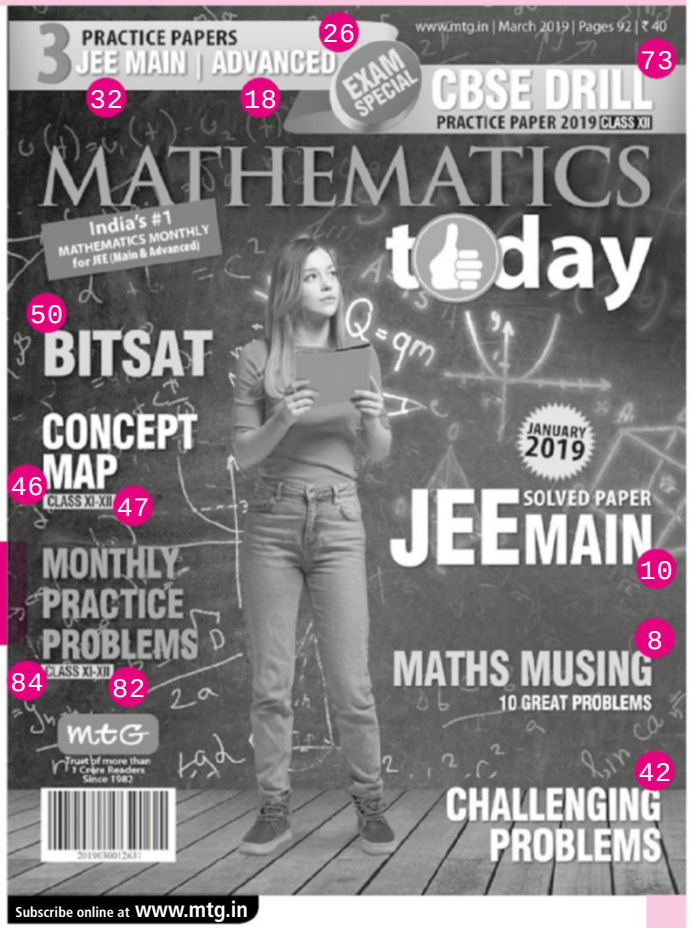

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During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.
Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM Set 195

JEE MAIN

1. Domain of the function $f(x)$, if $3^x + 3^{f(x)}$ = minimum of $\phi(t)$, where $\phi(t)$ = minimum of $\{2t^3 - 15t^2 + 36t - 25, 2 + |\sin t| ; 2 \le t \le 4\}$ is
(a) $(-\infty, 1)$ (b) $(-\infty, \log_3 e)$
(c) $(0, \log_3 2)$ (d) $(-\infty, \log_3 2)$

2. If in a triangle ABC, $a = 5$, $b = 4$ and $\cos(A - B) = \frac{31}{32}$, then the third side c is equal to
(a) 3 (b) 6 (c) 7 (d) 9

3. The values of x for which the angle between the vectors $2x^2\hat{i} + 4x\hat{j} + \hat{k}$ and $7\hat{i} - 2\hat{j} + x\hat{k}$ are obtuse and the angle between the z-axis and $7\hat{i} - 2\hat{j} + x\hat{k}$ is acute and less than $\frac{\pi}{6}$ is given by
(a) $0 < x < \frac{1}{2}$ (b) $0 > \frac{1}{2}$ or $x < 0$
(c) $\frac{1}{2} < x < 15$
(d) there is no such value for x

4. A straight line touches the rectangular hyperbola $9x^2 - 9y^2 = 8$ and the parabola $y^2 = 32x$. An equation of the line is
(a) $9x + 3y - 8 = 0$ (b) $9x - 3y + 8 = 0$
(c) $9x + 3y + 4 = 0$ (d) $9x - 3y - 8 = 0$

5. $\int_0^{\pi/4} \sin x \, d(x - [x])$ is equal to (where $[\cdot]$ denotes the greatest integer function)
(a) 1/2 (b) $1 - \frac{1}{\sqrt{2}}$
(c) 1 (d) none of these

JEE ADVANCED

6. If $\theta = \tan^{-1}(2\tan^2\theta) - \tan^{-1}\left(\left(\frac{1}{3}\right)\tan\theta\right)$, then $\tan\theta$ is equal to
(a) –2 (b) 1 (c) $\frac{2}{3}$ (d) 2

COMPREHENSION

A is a set containing 10 elements. A subset P of A is chosen at random and the set A is reconstructed by replacing the elements of P. Another subset Q of A is now chosen at random. Then, the probability that

7. $P \cup Q = A$, is
(a) $\left(\frac{1}{2}\right)^{10}$ (b) $\left(\frac{2}{3}\right)^{10}$ (c) $\left(\frac{3}{4}\right)^{10}$ (d) $\left(\frac{2}{5}\right)^{10}$

8. $P \cap Q$ contains atleast 9 elements, is
(a) $\frac{15}{2^{20}}$ (b) $\frac{31}{2^{20}}$ (c) $\frac{30}{2^{20}}$ (d) $\frac{41}{2^{20}}$

NUMERICAL ANSWER TYPE

9. z_1 and z_2 are two complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular, while z_2 is not unimodular, then $|z_1|$ =

MATRIX MATCH

10. In the following $[x]$ denotes the greatest integer less than or equal to x

Column-I		Column-II	
(P)	$x\|x\|$	(1)	continuous in (–1, 1)
(Q)	$\sqrt{\|x\|}$	(2)	differentiable in (–1, 1)
(R)	$x + [x]$	(3)	strictly increasing in (–1, 1)
(S)	$\|x - 1\| + \|x + 1\|$	(4)	not differentiable at atleast one point in (–1, 1)

	P	Q	R	S
(a)	2, 3	3, 4	1, 2	2, 4
(b)	1, 2	3, 4	1, 3, 4	2, 3, 4
(c)	1, 2, 3	1, 4	3, 4	1, 2
(d)	2, 3	3, 4	1, 3, 4	2, 4

See Solution Set of Maths Musing 194 on page no. 49

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SOLVED PAPER

1. Let S and S' be the foci of an ellipse and B be any one of the extremities of its minor axis. If $\Delta S'BS$ is a right angled triangle with right angle at B and area $(\Delta S'BS) = 8$ sq. units, then the length of a latus rectum of the ellipse is

(a) 2 (b) 4 (c) $4\sqrt{2}$ (d) $2\sqrt{2}$

2. The expression $\sim(\sim p \rightarrow q)$ is logically equivalent to

(a) $p \wedge q$ (b) $\sim p \wedge q$ (c) $\sim p \wedge \sim q$ (d) $p \wedge \sim q$

3. If the sum of the first 15 terms of the series

$$\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + ...$$

is equal to 225 k, then k is equal to

(a) 27 (b) 9 (c) 108 (d) 54

4. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$; $\alpha, \beta \in [0, \pi]$, then $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal to

(a) -1 (b) 0 (c) $\sqrt{2}$ (d) $-\sqrt{2}$

5. In a game, a man wins Rs. 100 if he gets 5 or 6 on a throw of a fair die and lose Rs. 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is

(a) $\frac{400}{9}$ loss (b) $\frac{400}{3}$ gain

(c) 0 (d) $\frac{400}{3}$ loss

6. Let S be the set of all real values of λ such that a plane passing through the points $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ also passes through the point $(-1, -1, 1)$. Then S is equal to

(a) $\{1, -1\}$ (b) $\{3, -3\}$ (c) $\{\sqrt{3}\}$ (d) $\{\sqrt{3}, -\sqrt{3}\}$

7. The tangent to the curve $y = x^2 - 5x + 5$, parallel to the line $2y = 4x + 1$, also passes through the point

(a) $\left(\frac{7}{2}, \frac{1}{4}\right)$ (b) $\left(\frac{1}{4}, \frac{7}{2}\right)$

(c) $\left(-\frac{1}{8}, 7\right)$ (d) $\left(\frac{1}{8}, -7\right)$

8. The total number of irrational terms in the binomial expansion of $(7^{1/5} - 3^{1/10})^{60}$ is

(a) 55 (b) 54 (c) 48 (d) 49

9. If a straight line passing through the point $P(-3, 4)$ is such that its intercepted portion between the coordinate axes is bisected at P, then its equation is

(a) $4x + 3y = 0$ (b) $4x - 3y + 24 = 0$

(c) $3x - 4y + 25 = 0$ (d) $x - y + 7 = 0$

10. $\lim\limits_{x \to 1^-} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$ is equal to

(a) $\sqrt{\pi}$ (b) $\frac{1}{\sqrt{2\pi}}$ (c) $\sqrt{\frac{\pi}{2}}$ (d) $\sqrt{\frac{2}{\pi}}$

11. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P., then n can be

(a) 12 (b) 14 (c) 9 (d) 11

12. Let f be a differentiable function such that $f(1) = 2$ and $f'(x) = f(x)$ for all $x \in R$. If $h(x) = f(f(x))$, then $h'(1)$ is equal to

(a) $2e^2$ (b) $4e$ (c) $2e$ (d) $4e^2$

13. The set of all values of λ for which the system of linear equations

$x - 2y - 2z = \lambda x$

$x + 2y + z = \lambda y$

$-x - y = \lambda z$

has a non-trivial solution

(a) is an empty set

(b) contains exactly two elements

(c) is a singleton

(d) contains more than two elements

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14. The number of integral values of m for which the quadratic expression, $(1 + 2m)x^2 - 2(1 + 3m)x + 4(1 + m)$, $x \in R$, is always positive is

(a) 8 (b) 7 (c) 3 (d) 6

15. The mean and the variance of five observations are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4; then the absolute value of the difference of the other two observations is

(a) 3 (b) 1 (c) 7 (d) 5

16. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$, then $|\alpha - \beta|$ is equal to

(a) 45° (b) 60° (c) 90° (d) 30°

17. If the function f given by $f(x) = x^3 - 3(a - 2)x^2 + 3ax + 7$, for some $a \in R$ is increasing in $(0, 1]$ and decreasing in $[1, 5)$, then a root of the equation, $\frac{f(x)-14}{(x-1)^2} = 0$ $(x \neq 1)$ is

(a) 7 (b) –7 (c) 5 (d) 6

18. The integral $\int \frac{3x^{13} + 2x^{11}}{(2x^4 + 3x^2 + 1)^4} dx$ is equal to (where C is a constant of integration)

(a) $\frac{x^4}{6(2x^4 + 3x^2 + 1)^3} + C$ (b) $\frac{x^4}{(2x^4 + 3x^2 + 1)^3} + C$

(c) $\frac{x^{12}}{6(2x^4 + 3x^2 + 1)^3} + C$ (d) $\frac{x^{12}}{(2x^4 + 3x^2 + 1)^3} + C$

19. The integral $\int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^{x}\right\} \log_e x \, dx$ is equal to

(a) $\frac{1}{2} - e - \frac{1}{e^2}$ (b) $-\frac{1}{2} + \frac{1}{e} - \frac{1}{2e^2}$

(c) $\frac{3}{2} - e - \frac{1}{2e^2}$ (d) $\frac{3}{2} - \frac{1}{e} - \frac{1}{2e^2}$

20. If a curve passes through the point $(1, -2)$ and has slope of the tangent at any point (x, y) on it as $\frac{x^2 - 2y}{x}$, then the curve also passes through the point

(a) $(-\sqrt{2}, 1)$ (b) $(-1, 2)$ (c) $(\sqrt{3}, 0)$ (d) $(3, 0)$

21. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is

(a) 11 (b) 9 (c) 12 (d) 7

22. If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30° and the angle of depression of reflection of the cloud in the lake from P be 60°, then the height of the cloud (in metres) from the surface of the lake is

(a) 50 (b) 45 (c) 60 (d) 42

23. $\lim_{n\to\infty}\left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + \ldots + \frac{1}{5n}\right)$ is equal to

(a) $\tan^{-1}(2)$ (b) $\frac{\pi}{2}$ (c) $\tan^{-1}(3)$ (d) $\frac{\pi}{4}$

24. If a circle of radius R passes through the origin O and intersects the coordinate axes at A and B, then the locus of the foot of perpendicular from O on AB is

(a) $(x^2 + y^2)^3 = 4R^2x^2y^2$ (b) $(x^2 + y^2)^2 = 4Rx^2y^2$

(c) $(x^2 + y^2)(x + y) = R^2xy$ (d) $(x^2 + y^2)^2 = 4R^2x^2y^2$

25. If an angle between the line, $\frac{x+1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$ and the plane, $x - 2y - kz = 3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of k is

(a) $\sqrt{\frac{5}{3}}$ (b) $-\frac{3}{5}$ (c) $\sqrt{\frac{3}{5}}$ (d) $-\frac{5}{3}$

26. The equation of a tangent to the parabola, $x^2 = 8y$, which makes an angle θ with the positive direction of x-axis is

(a) $x = y\cot\theta - 2\tan\theta$ (b) $y = x\tan\theta - 2\cot\theta$

(c) $x = y\cot\theta + 2\tan\theta$ (d) $y = x\tan\theta + 2\cot\theta$

27. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the student selected has opted neither for NCC nor for NSS is

(a) $\frac{1}{6}$ (b) $\frac{5}{6}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

28. Let z_1 and z_2 be two complex numbers satisfying $|z_1| = 9$ and $|z_2 - 3 - 4i| = 4$. Then the minimum value of $|z_1 - z_2|$ is

(a) $\sqrt{2}$ (b) 2 (c) 0 (d) 1

29. If $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$; then for all $\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$, $\det(A)$ lies in the interval

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(a) $\left(\frac{3}{2}, 3\right]$ (b) $\left[\frac{5}{2}, 4\right)$ (c) $\left(1, \frac{5}{2}\right]$ (d) $\left(0, \frac{3}{2}\right]$

30. Let Z be the set of integers.

If $A = \{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$ and $B = \{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set $A \times B$ is

(a) 2^{18} (b) 2^{12} (c) 2^{15} (d) 2^{10}

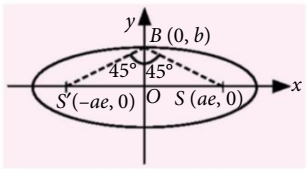
SOLUTIONS

1. (b) : Since $\angle S'BS = 90°$

$\therefore$ Slope of $S'B \times$ Slope of $SB = -1$

$\Rightarrow \frac{-b}{-ae} \times \frac{-b}{ae} = -1 \Rightarrow b^2 = a^2e^2$ ...(i)

Now, area of $\Delta S'BS = 8$



$\Rightarrow \frac{1}{2} \times 2ae \times b = 8$

$\Rightarrow ae \times b = 8$

$\Rightarrow a^2e^2 \times b^2 = 64$

$\Rightarrow (a^2e^2)^2 = 64$ [Using (i)]

$\Rightarrow a^2e^2 = 8 \Rightarrow ae = \pm 2\sqrt{2}$

$\therefore b = 2\sqrt{2}$

Again, $a^2 = a^2e^2 + b^2$ $\left[\because e = \sqrt{1 - \frac{b^2}{a^2}}\right]$

$\Rightarrow a^2 = 8 + 8 = 16 \Rightarrow a = \pm 4$

$\therefore$ Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 8}{4} = 4$

2. (c) :

p	$\sim p$	q	$\sim q$	$\sim p \to q$	$\sim(\sim p \to q)$	$(\sim p \wedge \sim q)$
T	F	T	F	T	F	F
F	T	T	F	T	F	F
T	F	F	T	T	F	F
F	T	F	T	F	T	T

Above truth table shows that $\sim(\sim p \to q)$ is logically equivalent to $(\sim p \wedge \sim q)$.

3. (a) : Let

$S = \left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + \ldots$ 15 terms

$= \left(\frac{3}{4}\right)^3 + \left(\frac{3}{2}\right)^3 + \left(\frac{9}{4}\right)^3 + 3^3 + \left(\frac{15}{4}\right)^3 + \ldots$ 15 terms

$= \left(\frac{3}{4}\right)^3 + \left(\frac{6}{4}\right)^3 + \left(\frac{9}{4}\right)^3 + \left(\frac{12}{4}\right)^3 + \left(\frac{15}{4}\right)^3 + \ldots$ 15 terms

$= \frac{27}{64}\sum_{x=1}^{15} x^3 = \frac{27}{64}\left[\frac{15(15+1)}{2}\right]^2 = 27 \times 225$

$\therefore 225\,k = 27 \times 225 \Rightarrow k = 27$

4. (d) : Since, A.M. $\geq$ G.M.

$\therefore \frac{\sin^4\alpha + 4\cos^4\beta + 1 + 1}{4} \geq (\sin^4\alpha \cdot 4\cos^4\beta \cdot 1 \cdot 1)^{1/4}$

$\Rightarrow \sin^4\alpha + 4\cos^4\beta + 2 \geq 4\sqrt{2}\sin\alpha\cos\beta$

But according to question, we have

$\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$

$\Rightarrow$ A.M. = G.M. $\Rightarrow \sin^4\alpha = 1 = 4\cos^4\beta$

$\Rightarrow \sin\alpha = 1, \cos\beta = \pm\frac{1}{\sqrt{2}}$

$\therefore \sin\alpha = 1$ and $\sin\beta = \frac{1}{\sqrt{2}}$ as $\alpha, \beta \in [0, \pi]$

Now, $\cos(\alpha + \beta) - \cos(\alpha - \beta) = -2\sin\alpha\sin\beta$

$= -2 \cdot 1 \cdot \frac{1}{\sqrt{2}} = -\sqrt{2}$

5. (c) : Let A = Probability that outcome is 5 or 6 = $\frac{1}{3}$

B = Probability that outcome is other than 5 or 6 = $\frac{2}{3}$

$\therefore$ Expected gain/loss

$= A \times 100 + BA(-50 + 100) + B^2A(-50 - 50 + 100) + B^3(-150)$

$= \frac{1}{3} \times 100 + \frac{2}{3} \cdot \frac{1}{3}(50) + \frac{4}{9} \cdot \frac{1}{3}(0) + \left(\frac{2}{3}\right)^3(-150)$

$= \frac{100}{3} + \frac{100}{9} - \frac{8}{27} \times 150 = 0$

6. (d) : Equation of plane passing through the points $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ is

$$\begin{vmatrix} x + \lambda^2 & y - 1 & z - 1 \\ 1 + \lambda^2 & -\lambda^2 - 1 & 0 \\ 1 + \lambda^2 & 0 & -\lambda^2 - 1 \end{vmatrix} = 0$$

$\Rightarrow (x + \lambda^2)(\lambda^2 + 1)^2 + (y - 1)(1 + \lambda^2)^2 + (z - 1)(1 + \lambda^2)^2 = 0$

$\Rightarrow (1 + \lambda^2)^2[(x + \lambda^2) + (y - 1) + (z - 1)] = 0$

Since, this plane is also passes through the point $(-1, -1, 1)$.

$\therefore (1 + \lambda^2)^2[(-1 + \lambda^2) + (-1 - 1)] = 0$

$\Rightarrow (1 + \lambda^2)^2(\lambda^2 - 3) = 0$

So, real values of λ are $\pm\sqrt{3}$.

$\therefore S = \{\sqrt{3}, -\sqrt{3}\}$

7. (d) : Here, curve is $y = x^2 - 5x + 5$

$\Rightarrow \frac{dy}{dx} = 2x - 5$

Since, the tangent is parallel to the line $2y = 4x + 1$

$\therefore \frac{dy}{dx} = 2x - 5 = 2$

$\Rightarrow x = \frac{7}{2}$

When $x = \frac{7}{2}$, $y = \left(\frac{7}{2}\right)^2 - 5 \times \frac{7}{2} + 5 = \frac{-1}{4}$

$\therefore$ Equation of tangent at $\left(\frac{7}{2}, \frac{-1}{4}\right)$ is

$$\left(y+\frac{1}{4}\right)=2\left(x-\frac{7}{2}\right) \Rightarrow y-2x+\frac{29}{4}=0$$

Only the point in option (d) *i.e.*, $\left(\frac{1}{8}, -7\right)$ satisfies the above equation.

8. (b) : Given expression is $(7^{1/5} - 3^{1/10})^{60}$

General term of this expansion is

$$T_{r+1} = (-1)^r \, {}^{60}C_r 7^{\frac{60-r}{5}} 3^{\frac{r}{10}}$$

For rational terms, $r = 0, 10, 20, 30, 40, 50, 60$

So, number of rational terms = 7.

$\therefore$ Number of irrational terms = 61 – 7 = 54.

9. (b) : Let the equation of the line is $\frac{x}{a}+\frac{y}{b}=1$

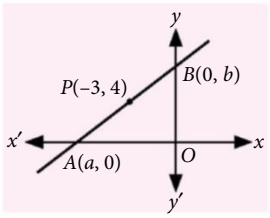
$\therefore \quad \frac{a}{2}=-3$ and $\frac{b}{2}=4$

$\Rightarrow \quad a=-6$ and $b=8$

So, equation of the line is

$\frac{x}{-6}+\frac{y}{8}=1$

$\Rightarrow \quad 4x - 3y + 24 = 0$



10. (d) : We have, $\lim\limits_{x\to1^-} \frac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$

$$= \lim_{x\to1^-} \frac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}} \times \frac{\sqrt{\pi}+\sqrt{2\sin^{-1}x}}{\sqrt{\pi}+\sqrt{2\sin^{-1}x}}$$

$$= \lim_{x\to1^-} \frac{2\left(\frac{\pi}{2}-\sin^{-1}x\right)}{\sqrt{1-x}\left(\sqrt{\pi}+\sqrt{2\sin^{-1}x}\right)} = \lim_{x\to1^-} \frac{2\cos^{-1}x}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{\pi}}$$

$$= \lim_{\theta\to0^+} \frac{2\theta}{\sqrt{2}\sin\left(\frac{\theta}{2}\right)} \cdot \frac{1}{2\sqrt{\pi}} \qquad [\text{Putting } x = \cos\theta]$$

$$= \frac{4}{2\sqrt{2\pi}} = \sqrt{\frac{2}{\pi}}$$

11. (b) : Since, ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P.

$\therefore \quad 2 \times {}^nC_5 = {}^nC_4 + {}^nC_6$

$$\Rightarrow \quad 2\times\frac{\lfloor n}{\lfloor 5 \lfloor n-5}=\frac{\lfloor n}{\lfloor 4 \lfloor n-4}+\frac{\lfloor n}{\lfloor 6 \lfloor n-6}$$

$$\Rightarrow \quad \frac{2}{5(n-5)}=\frac{1}{(n-4)(n-5)}+\frac{1}{6\times5}$$

$$\Rightarrow \quad \frac{2}{5}(n-4)=1+\frac{(n-4)(n-5)}{6\times5}$$

$\Rightarrow \quad 12(n - 4) = 30 + n^2 - 9n + 20$

$\Rightarrow \quad 21n - n^2 - 98 = 0 \Rightarrow n^2 - 21n + 98 = 0$

$\Rightarrow \quad (n - 14)(n - 7) = 0 \Rightarrow n = 7, 14$

But only $n = 14$ lies in the options.

12. (b) : Given, $\frac{f'(x)}{f(x)} = 1 \quad \forall x \in R$

Integrating both sides, we get

$\log f(x) = x + C \Rightarrow f(x) = e^{x+C} = e^x \cdot e^C$

$\because \quad f(1) = 2 \Rightarrow f(1) = e \cdot e^C = 2$

$$\Rightarrow \quad e^C = \frac{2}{e} \Rightarrow C = \log\frac{2}{e} \quad \therefore \; f(x) = e^x \cdot \frac{2}{e} = 2e^{x-1}$$

$\Rightarrow \quad f'(x) = 2e^{x-1}$

Now, $h(x) = f(f(x))$

$\therefore \quad h'(x) = f'(f(x)) \cdot f'(x)$

$\therefore \quad h'(1) = f'(f(1)) \cdot f'(1) = f'(2) f'(1)$

$= 2e \cdot 2 = 4e$

13. (c) : The given system of linear equations is

$(1 - \lambda)x - 2y - 2z = 0$

$x + (2 - \lambda)y + z = 0$

$-x - y - \lambda z = 0$

Since, it has a non-trivial solution.

$$\therefore \quad \begin{vmatrix} 1-\lambda & -2 & -2 \\ 1 & 2-\lambda & 1 \\ -1 & -1 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (1 - \lambda)[(2 - \lambda)(-\lambda) + 1] + 2(-\lambda + 1)$

$\qquad\qquad - 2(- 1 + 2 - \lambda) = 0$

$\Rightarrow \quad (1 - \lambda)[-2\lambda + \lambda^2 + 1 + 2 - 2] = 0$

$\Rightarrow \quad (1 - \lambda)(1 - \lambda)^2 = 0 \Rightarrow (1 - \lambda)^3 = 0 \Rightarrow \lambda = 1$

14. (b) : The given quadratic expression

$(1 + 2m)x^2 - 2(1+ 3m)x + 4(1 + m), x \in R$

is always positive if

$1 + 2m > 0 \quad \Rightarrow \quad m > \frac{-1}{2}$ ...(i)

And, $D < 0$

$\Rightarrow \quad 4(1 + 3m)^2 - 4(1 + 2m) \times 4(1 + m) < 0$

$\Rightarrow \quad (1 + 9m^2 + 6m) - 4(1 + 3m + 2m^2) < 0$

$\Rightarrow \quad m^2 - 6m - 3 < 0$

$\Rightarrow \quad 3-\sqrt{12} < m < 3+\sqrt{12}$ ...(ii)

From (i) and (ii), common interval is

$3-\sqrt{12} < m < 3+\sqrt{12}$

So, integral values of m are 0, 1, 2, 3, 4, 5, 6.

15. (c) : Let the other two observations are x_1 and x_2.

$\therefore \quad$ Mean $(\bar{x}) = \frac{3+4+4+x_1+x_2}{5} = 4$

$\Rightarrow \quad x_1 + x_2 = 9$ ...(i)

And variance $(\sigma^2) = \frac{\Sigma x_i^2}{n} - (\bar{x})^2$

$$\Rightarrow \quad 5.20 = \frac{9+16+16+x_1^2+x_2^2}{5} - 16$$

$\Rightarrow \quad \frac{41+x_1^2+x_2^2}{5}=21.2 \quad \Rightarrow \quad x_1^2+x_2^2=106-41$

$\Rightarrow \quad x_1^2+x_2^2=65$ ...(ii)

Using (i) and (ii), we get

$(x_1+x_2)^2=81 \Rightarrow 65+2x_1x_2=81 \Rightarrow 2x_1x_2=16$

$\therefore \quad (x_1-x_2)^2=65-16=49 \Rightarrow |x_1-x_2|=7$

16. (d) : Here, $\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}\vec{b}$

$\Rightarrow \quad (\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}=\frac{1}{2}\vec{b}$

Since, $\vec{b}$ and $\vec{c}$ are non-parallel vectors.

$\therefore \quad \vec{a}\cdot\vec{c}=\frac{1}{2}$ and $\vec{a}\cdot\vec{b}=0$

Since $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors.

$\therefore \quad \cos\beta=\frac{1}{2}$ and $\cos\alpha=0$

$\Rightarrow \quad \beta=60°$ and $\alpha=90° \quad \therefore \quad |\alpha-\beta|=30°$

17. (a) : Here, $f(x)=x^3-3(a-2)x^2+3ax+7$

$\therefore \quad f'(x)=3x^2-6(a-2)x+3a$

Since $f'(x)\geq 0 \ \forall\ x\in(0,1]$ and $f'(x)\leq 0 \ \forall\ x\in[1,5)$

$\therefore \quad f'(x)=0$ at $x=1$

$\Rightarrow \quad 3-6(a-2)+3a=0 \Rightarrow -3a+15=0 \Rightarrow a=5$

So, $f(x)=x^3-9x^2+15x+7$

Now, $\frac{f(x)-14}{(x-1)^2}=0 \quad \Rightarrow \quad \frac{x^3-9x^2+15x-7}{(x-1)^2}=0$

$\Rightarrow \quad \frac{(x-1)^2(x-7)}{(x-1)^2}=0 \quad \Rightarrow \quad x=7$

18. (c) : Let $I=\int\frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}dx=\int\frac{\frac{3}{x^3}+\frac{2}{x^5}}{\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^4}dx$

Now, put $2+\frac{3}{x^2}+\frac{1}{x^4}=z$

$\therefore \quad \left(\frac{-6}{x^3}-\frac{4}{x^5}\right)dx=dz \quad \Rightarrow \quad \left(\frac{3}{x^3}+\frac{2}{x^5}\right)dx=\frac{-dz}{2}$

$\therefore \quad I=\frac{-1}{2}\int\frac{dz}{z^4}=\frac{-1}{2}\left(\frac{z^{-3}}{-3}\right)+C$

$=\frac{1}{6z^3}+C=\frac{1}{6\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^3}+C$

$=\frac{x^{12}}{6(2x^4+3x^2+1)^3}+C$

19. (c) : Let $I=\int_1^e\left\{\left(\frac{x}{e}\right)^{2x}-\left(\frac{e}{x}\right)^x\right\}\log_e x\,dx$

$=\int_1^e\left(\frac{x}{e}\right)^{2x}\log_e x\,dx-\int_1^e\left(\frac{e}{x}\right)^x\log_e x\,dx$

Let $\left(\frac{x}{e}\right)^{2x}=u$ and $\left(\frac{e}{x}\right)^x=v$

When $x=1$, $u=\left(\frac{1}{e}\right)^2$ and $v=e$

And when $x=e$, $u=1$ and $v=1$

Now, $2x\log\left(\frac{x}{e}\right)=\log u$

$\Rightarrow \quad 2\left[\log\left(\frac{x}{e}\right)+x\frac{e}{x}\cdot\frac{1}{e}\right]dx=\frac{du}{u}$

$\Rightarrow \quad 2[\log x-1+1]dx=\frac{du}{u} \quad \Rightarrow \quad \log x\,dx=\frac{1}{2}\cdot\frac{du}{u}$

and $x\log\left(\frac{e}{x}\right)=\log v \quad \Rightarrow \quad x(1-\log x)=\log v$

$\Rightarrow \quad x-x\log x=\log v$

$\Rightarrow \quad \left(1-\log x-x\cdot\frac{1}{x}\right)dx=\frac{dv}{v} \quad \Rightarrow \quad -\log x\,dx=\frac{dv}{v}$

$\therefore \quad I=\frac{1}{2}\int_{\left(\frac{1}{e}\right)^2}^{1}u\frac{du}{u}+\int_e^1 v\frac{dv}{v}=\frac{3}{2}-e-\frac{1}{2e^2}$

20. (c) : Here, $\frac{dy}{dx}=\frac{x^2-2y}{x}$

$\Rightarrow \quad \frac{dy}{dx}+\frac{2}{x}y=x$

$\therefore \quad \text{I.F.}=e^{\int\frac{2}{x}dx}=x^2$

$\therefore \quad \text{Solution is } y\cdot x^2=\int x\cdot x^2\,dx+C$

$\Rightarrow \quad yx^2=\frac{x^4}{4}+C$

Since, the curve passes through the point (1, –2).

$\therefore \quad -2=\frac{1}{4}+C \quad \Rightarrow \quad C=\frac{-9}{4}$

$\therefore \quad yx^2=\frac{x^4}{4}-\frac{9}{4} \quad \Rightarrow x^4-4x^2y-9=0$

Now, only the point in option (c) *i.e.*, $(\sqrt{3}, 0)$ satisfies the above equation.

21. (c) : There are m men and 2 women.

So, number of games played by the men between themselves $= {}^mC_2\times 2$

And number of games played between men and women $= {}^mC_1\,{}^2C_1\times 2$

$\therefore \quad {}^mC_2\times 2={}^mC_1\,{}^2C_1\times 2+84$

$\Rightarrow \quad m(m-1)=m\times 2\times 2+84$

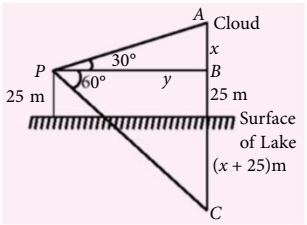
$\Rightarrow \quad m^2-5m-84=0 \Rightarrow (m-12)(m+7)=0$

$\Rightarrow \quad m=12$ or -7

$\because$ m can't be negative, so $m=12$.

22. (a) : In ΔPAB,

$\tan 30° = \frac{x}{y} \Rightarrow y = \sqrt{3}x$ ...(i)



In ΔPBC,

$\tan 60° = \frac{25 + x + 25}{y}$

$\Rightarrow \quad \sqrt{3}y = 50 + x$

$\Rightarrow \quad 3x = 50 + x$ [Using (i)] $\Rightarrow \quad x = 25$

$\therefore$ Height of cloud from the surface of the lake $= 25 + 25 = 50$ m

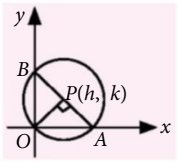
23. (a) : Given, $\lim\limits_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+...+\frac{1}{5n}\right)$

$= \lim\limits_{n\to\infty}\sum\limits_{r=1}^{2n}\frac{n}{n^2+r^2} = \lim\limits_{n\to\infty}\sum\limits_{r=1}^{2n}\frac{1}{n\left(1+\frac{r^2}{n^2}\right)}$

$= \int\limits_0^2\left(\frac{1}{1+x^2}\right)dx$ [Putting $\frac{r}{n} = x$, we get $\frac{1}{n}dr = dx$ and $r = 1 \Rightarrow x = 0$, $r = 2n \Rightarrow x = 2$]

$= \left[\tan^{-1}x\right]_0^2 = \tan^{-1}2$

24. (a) : Let $P(h, k)$ be the foot of the perpendicular from O to AB.



$\therefore \quad$ Slope of $AB = \frac{-h}{k}$

$\therefore$ Equation of AB is $hx + ky = h^2 + k^2$

$\Rightarrow \quad \frac{x}{\frac{h^2+k^2}{h}} + \frac{y}{\frac{h^2+k^2}{k}} = 1$

So, coordinates of A and B are

$\left(\frac{h^2+k^2}{h}, 0\right)$ and $\left(0, \frac{h^2+k^2}{k}\right)$ respectively.

Now, $AB = 2R \Rightarrow (AB)^2 = 4R^2$

$\Rightarrow \quad \frac{(h^2+k^2)^2}{h^2} + \frac{(h^2+k^2)^2}{k^2} = 4R^2$

$\Rightarrow \quad (h^2 + k^2)^3 = 4R^2h^2k^2$

Thus, locus of P is $(x^2 + y^2)^3 = 4R^2x^2y^2$.

25. (a) : D.R.'s of line are 2, 1, −2

and normal vector to the plane is $\hat{i} - 2\hat{j} - k\hat{k}$.

Let α be the angle between the line and the plane.

So, $\sin\alpha = \frac{(2\hat{i} + \hat{j} - 2\hat{k})\cdot(\hat{i} - 2\hat{j} - k\hat{k})}{3\sqrt{1+4+k^2}}$

$\Rightarrow \quad \sin\alpha = \frac{2k}{3\sqrt{k^2+5}}$ ...(i) $\cos\alpha = \frac{2\sqrt{2}}{3}$ [Given]...(ii)

$\because \quad \sin^2\alpha + \cos^2\alpha = 1$

$\Rightarrow \quad \frac{4k^2}{9(k^2+5)} + \frac{8}{9} = 1$ [Using (i) & (ii)]

$\Rightarrow \quad \frac{4k^2}{k^2+5} + 8 = 9 \Rightarrow 4k^2 = k^2 + 5$

$\Rightarrow \quad k^2 = \frac{5}{3} \Rightarrow k = \sqrt{\frac{5}{3}}$

26. (c) : Given equation of parabola is $x^2 = 8y$

$\therefore \quad \frac{8dy}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{x}{4} = \tan\theta$ (Given)

$\Rightarrow \quad x = 4\tan\theta \quad \therefore \quad y = 2\tan^2\theta$

Now, equation of tangent at $(4\tan\theta, 2\tan^2\theta)$ is

$y - 2\tan^2\theta = \tan\theta\,(x - 4\tan\theta)$

$\Rightarrow \quad x = y\cot\theta + 2\tan\theta$

27. (a) : Let C and S represents the set of students who opted for NCC and NSS respectively.

$\therefore \quad n(C) = 40$, $n(S) = 30$, $n(C \cap S) = 20$

Now, $n(C \cup S) = 40 + 30 - 20 = 50$

So, number of students who has opted neither for NCC nor for NSS are $60 - 50 = 10$

So, P(Neither C nor S) $= \frac{10}{60} = \frac{1}{6}$

28. (c) : Given, $|z_1| = 9$, represents a circle with centre $C_1(0, 0)$ and radius $r_1 = 9$.

Also, $|z_2 - (3 + 4i)| = 4$ represents a circle with centre $C_2(3, 4)$ and radius $r_2 = 4$.

$\therefore \quad C_1C_2 = |r_1 - r_2| = 5$

So, circles touches each other internally.

$\therefore \quad |z_1 - z_2|_{\min} = 0$

29. (a) : Here, $\det(A) = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$

$= (1 + \sin^2\theta) - \sin\theta\,(0) + 1(\sin^2\theta + 1) = 2(1 + \sin^2\theta)$

$\because \theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \Rightarrow \sin\theta \in \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\Rightarrow \quad \sin^2\theta \in \left[0, \frac{1}{2}\right) \quad \therefore \quad \det(A) \in [2, 3)$

30. (c) : Given, $A = \{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$

$\therefore \quad 2^{(x+2)(x^2-5x+6)} = 2^0$

$\Rightarrow \quad (x + 2)(x^2 - 5x + 6) = 0$

$\Rightarrow \quad (x + 2)(x - 2)(x - 3) = 0$

$\Rightarrow \quad x = 2, -2, 3 \quad \therefore \quad A = \{-2, 2, 3\}$

Also, $B = \{x \in z : -3 < 2x - 1 < 9\}$

So, $-3 < 2x - 1 < 9 \Rightarrow -2 < 2x < 10 \Rightarrow -1 < x < 5$

$\therefore \quad B = \{0, 1, 2, 3, 4\}$

Thus, $n(A \times B) = 15$

So, number of subsets of $A \times B$ is 2^{15}.

JEE WORK OUTS

JEE Advanced Exam on 19th May, 2019

One or More Than One Option(s) Correct Type

1. Let $S=\left\{x\in(-\pi,\pi): x\neq 0,\pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}\sec x+\text{cosec}\, x+2(\tan x-\cot x)=0$ in the set S is equal to

(a) $\frac{-7\pi}{9}$ (b) $\frac{-2\pi}{9}$ (c) 0 (d) $\frac{5\pi}{9}$

2. Let A and B be two 3×3 matrices of real numbers, where A is symmetric and B is skew symmetric. If $(A+B)(A-B)=(A-B)(A+B)$ and $(AB)^T=(-1)^k AB$, then the possible value(s) of k is/are

(a) 2 (b) 3 (c) 5 (d) 6

3. If the area enclosed by the parabola $y=1+x^2$ and a normal drawn to it with gradient -1, is A then $3A$ is

(a) 2 (b) 3 (c) 4 (d) 5

4. The number of 5 cards combinations out of a deck of 52 cards if there is exactly one ace in each combination is

(a) 768320 (b) 777320 (c) 778230 (d) 778320

5. If $2\times 2^2+3\times 2^3+4\times 2^4+\ldots+n\times 2^n=2^{n+10}$, then the value of n is

(a) 513 (b) 512 (c) 511 (d) 520

6. If $f(x)=\begin{cases}3-x^2, & x\leq 2\\ \sqrt{a+14}-|x-48|, & x>2\end{cases}$ and if $f(x)$ has a local maximum at $x=2$, then greatest value of a is

(a) 2013 (b) 2012 (c) 2011 (d) 2010

7. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true?

(a) $3a^2-26a+55=0$ (b) $3a^2-32a+84=0$
(c) $3a^2-34a+91=0$ (d) $3a^2-23a+44=0$

8. If $y=\sin x-\cos x$ and $f(x)=\frac{d^{17}y}{dx^{17}}$, then the value of $\left(\frac{1024}{\sqrt{2}}\right)f\left(\frac{\pi}{4}\right)$ must be

(a) 1024 (b) 1025 (c) 1022 (d) 1026

9. If $\tan^{-1}y=4\tan^{-1}x$, then y is infinite if

(a) $x^2=3+2\sqrt{2}$ (b) $x^2=3-2\sqrt{2}$
(c) $x^4=6x^2-1$ (d) $x^4=6x^2+1$

10. In a ΔABC, $\frac{a}{b}=2+\sqrt{3}$ and $\angle C=60°$. Then the ordered pair $(\angle A, \angle B)$ is equal to

(a) (15°, 105°) (b) (105°, 15°)
(c) (45°, 75°) (d) (75°, 45°)

11. For which values of 'a' will the function $f(x)=x^4+ax^3+\frac{3x^2}{2}+1$ be concave upward along the entire real line ?

(a) $a\in[0,\infty)$ (b) $a\in[-2,\infty)$
(c) $a\in[-2,2]$ (d) $a\in(0,\infty)$

12. If $\int\frac{dx}{\cos(x-a)\cos(x-b)}=\frac{1}{A}[\log_e(f(x))/\log_e g(x)]+c$, then

(a) $A=\sin(a-b)$ (b) $f(x)=\sec(x-b)$
(c) $g(x)=\sec(x-a)$ (d) $A=\sin(b-a)$

13. Let $f(x)=\cos(\pi(|x|+2[x]))$, where $[\cdot]$ represents greatest integer function. Then

(a) $f(x)$ is neither odd nor even.
(b) range of $f(x)$ is $[-1, 1]$.
(c) $f(x)=|f(x)|$ for all x.
(d) none of these

14. The value of a, for which the points A, B, C with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$, $a\hat{i}-3\hat{j}+\hat{k}$

respectively are the vertices of a right angled triangle with $C=\frac{\pi}{2}$ are

(a) 1 (b) 2 (c) – 1 (d) – 2

15. If $f(x)=\frac{e^{2x}-(1+4x)^{1/2}}{\ln(1-x^2)}$ for $x \neq 0$, then f has

(a) an irremovable discontinuity at $x = 0$

(b) a removable discontinuity at $x = 0$ and $f(0) = \frac{1}{4}$

(c) a removable discontinuity at $x = 0$ and $f(0)=-\frac{1}{4}$

(d) none of these

16. The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix} = 0$ if

(a) a, b, c are in A.P.

(b) a, b, c are in G.P.

(c) α is a root of $ax^2 + 2bx + c = 0$

(d) $(x - \alpha)$ is a factor of $ax^2 + 2bx + c = 0$

17. If 2 and 31 appear as two terms in an A.P., then

(a) common difference of the A.P. is a rational number

(b) all the terms of the A.P. must be rational.

(c) all the terms of the A.P. must be integers.

(d) sum to any finite number of terms of the A.P. must be rational.

18. If $f(x) = x^3 - x^2 + 100x + 2002$, then

(a) $f(1000) > f(1001)$ (b) $f\left(\frac{1}{2000}\right) > f\left(\frac{1}{2001}\right)$

(c) $f(x - 1) > f(x - 2)$ (d) none of these

19. Three lines $px + qy + r = 0$, $qx + ry + p = 0$ and $rx + py + q = 0$ are concurrent, if

(a) $p + q + r = 0$

(b) $p^2 + q^2 + r^2 = pq + qr + rp$

(c) $p^3 + q^3 + r^3 = 3pqr$

(d) none of these

20. Consider the points $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0, 0, c)$, where $abc \neq 0$; then

(a) the equation of plane ABC is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

(b) the area of ΔABC is $\frac{1}{2}\sqrt{b^2c^2+c^2a^2+a^2b^2}$

(c) the equation of the plane ABC is $ax + by + cz = 1$

(d) none of these

Comprehension Type

Paragraph for Q. No. 21 to 23

Consider the quadratic polynomial
$f(x) = x^2 - 4ax + 5a^2 - 6a$, $(a \in R)$.

21. The value of a for which roots of $f(x) = 0$ are equal in magnitude and opposite in sign, is

(a) 0 (b) 1

(c) 2 (d) none of these

22. Number of values of a for which the equation $f(x) = 0$ has one root equal to zero, is

(a) 0 (b) 1 (c) 2 (d) 3

23. The largest integral value of 'a' for which range of $f(x)$ is $[-5, \infty)$ for every real x, is

(a) 1 (b) 5 (c) 7 (d) 10

Paragraph for Q. No. 24 to 26

Consider the function $f(x)$, a fourth degree polynomial such that $\lim_{x\to -1}\frac{f(x)}{(x+1)^3} = 1$ and $f'''(0) = 0$

24. The maximum value of $f(x)$ is

(a) $\frac{13}{4}$ (b) $\frac{17}{4}$ (c) $\frac{23}{4}$ (d) $\frac{27}{4}$

25. The subnormal of the curve $y = f(x)$ where it cuts the y-axis is

(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) $\frac{1}{2}$

26. The number of inflection points of the curve $y = f(x)$ is

(a) 0 (b) 1 (c) 2 (d) 3

Paragraph for Q. No. 27 to 29

Consider the lines $L_1 : \frac{x+1}{3}=\frac{y+2}{1}=\frac{z+1}{2}, L_2 : \frac{x-2}{1}=\frac{y+2}{2}=\frac{z-3}{3}$

27. The unit vector perpendicular to both L_1 and L_2 is

(a) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (b) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(c) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (d) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

28. The shortest distance between L_1 and L_2 is

(a) 0 (b) $\frac{17}{\sqrt{3}}$ (c) $\frac{41}{5\sqrt{3}}$ (d) $\frac{17}{5\sqrt{3}}$

29. The distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both L_1 and L_2 is

(a) $\frac{2}{5\sqrt{3}}$ (b) $\frac{7}{5\sqrt{3}}$ (c) $\frac{13}{5\sqrt{3}}$ (d) $\frac{23}{5\sqrt{3}}$

Matrix Match Type

30. Match the following.

Column-I		Column-II	
(A)	If sum of the coefficient of the first, second and third terms in the expansion of $\left(x^2+\frac{1}{x}\right)^n$ is 46, then coefficient of the term that does not contain x is	(p)	3
(B)	For a positive integer n, if the expansion of $\left(\frac{5}{x^2}+x^4\right)^n$ has a term independent of x, then minimum value of n can be	(q)	15
(C)	If in the expansion of $\left(x^3-\frac{1}{x^2}\right)^n$, $n \in N$, sum of the coefficients of x^5 and x^{10} is zero, then the value of n is	(r)	19
(D)	The coefficient of x^4 in the expansion of $(1 + x + x^2)^4$ is	(s)	84

(a) (A) → (p) ; (B) → (r) ; (C) → (q) ; (D) → (s)
(b) (A) → (q) ; (B) → (r) ; (C) → (p) ; (D) → (s)
(c) (A) → (s) ; (B) → (p) ; (C) → (q) ; (D) → (r)
(d) (A) → (r) ; (B) → (q) ; (C) → (p) ; (D) → (s)

31. Consider the circles C_1 of radius a and C_2 of radius b, $b > a$ both lying in the first quadrant and touching the coordinate axes. Find the value of b/a if

Column-I		Column-II	
(A)	C_1 and C_2 touch each other	(p)	$2+\sqrt{2}$
(B)	C_1 and C_2 are orthogonal	(q)	3
(C)	C_1 and C_2 intersect so that the common chord is longest	(r)	$2+\sqrt{3}$
(D)	C_2 passes through the centre of C_1	(s)	$3+2\sqrt{2}$

(a) (A) → (s) ; (B) → (p,s) ; (C) → (q) ; (D) → (q)
(b) (A) → (r) ; (B) → (q) ; (C) → (p,s) ; (D) → (q)
(c) (A) → (p,q) ; (B) → (r) ; (C) → (s) ; (D) → (p)
(d) (A) → (s) ; (B) → (r) ; (C) → (q) ; (D) → (p)

32. Match the following.

Column-I		Column-II	
(A)	$\lim_{x\to 2^+}\{x\}\frac{\sin(x-2)}{(x-2)^2}$ =	(p)	– 3
(B)	$\lim_{x\to 2}[2-x]+[x-2]-x$ =	(q)	1
(C)	$\lim_{x\to 2}\{2-x\}+\{x-2\}$ =	(r)	– 1
(D)	$\lim_{x\to 0^+}\frac{e^{\frac{1}{x}}-e^{\frac{-1}{x}}}{e^{\frac{1}{x}}+e^{\frac{-1}{x}}}$ =	(s)	2

(a) (A) → (q) ; (B) → (p) ; (C) → (q); (D) → (q)
(b) (A) → (p,r) ; (B) → (r) ; (C) → (p); (D) → (q)
(c) (A) → (p) ; (B) → (q) ; (C) → (r,s); (D) → (p)
(d) (A) → (q) ; (B) → (p,q) ; (C) → (r); (D) → (s)

Numerical Answer Type

33. Let $f: R^+ \to R$ be a function which satisfies $f(x)\cdot f(y)=f(xy)+2\left(\frac{1}{x}+\frac{1}{y}+1\right)$ for $x, y > 0$. Then possible value of $f(1/2)$ is

34. Water is dropped at the rate of 2 m^3/s into a cone of semi vertical angle 45°. If the rate at which periphery of water surface changes when the height of the water in the cone is 2 m is d, then the value of 5 d is

35. If $\sin\left\{2\left(\sin^{-1}\frac{\sqrt{5}}{3}-\cos^{-1}\frac{\sqrt{5}}{3}\right)\right\}$ is equal to $\frac{k\sqrt{5}}{81}$, then k =

36. If $g(x)$ is a differentiable function $g(1) \neq 0, g(-1) \neq 0$ and Rolle's theorem is not applicable to $f(x) = \frac{x^2-1}{g(x)}$ in $[-1, 1]$, then $g(x)$ has atleast k root in $(-1, 1)$, then k =

37. The greatest possible number of points of intersection of 8 straight lines and 4 circles is

38. If $\int (x^9+x^6+x^3)(2x^6+3x^3+6)^{1/3}dx = \frac{1}{A}(2x^9+3x^6+6x^3)^B+K$, then the value of $\frac{AB}{4}$ is

39. If the mean and the variance of a binomial variate X are 2 and 1 respectively, then 16 $P(X \geq 1)$ =

40. Let $\vec{a}=-\hat{i}-\hat{k}, \vec{b}=-\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}+2\hat{j}+3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}\cdot\vec{a}=0$, then $\vec{r}\cdot\vec{b}$ =

SOLUTIONS

1. (c) : $\sqrt{3}\sin x+\cos x=2\cos 2x$

$\Rightarrow \frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x=\cos 2x \Rightarrow \cos\left(x-\frac{\pi}{3}\right)=\cos 2x$

We have, $2x\pm\left(x-\frac{\pi}{3}\right)=2k\pi, k\in Z$

Then $x=(6k-1)\frac{\pi}{3}$ or $(6k+1)\frac{\pi}{9}, k\in Z$

The values of x in $(-\pi, \pi)$ are $-\frac{\pi}{3}, \frac{\pi}{9}, \frac{7\pi}{9}, -\frac{5\pi}{9}$

$\therefore$ Required sum = 0

2. (b, c) : Since A is symmetric, $A^T = A$.

Since B is skew-symmetric, $B^T = -B$.

Given, $(A + B)(A - B) = (A - B)(A + B)$

$\Rightarrow A^2 - AB + BA - B^2 = A^2 + AB - BA - B^2$

$\Rightarrow 2BA = 2AB \Rightarrow AB = BA$

Also, $(AB)^T = (-1)^k AB$

$\Rightarrow B^TA^T = (-1)^k AB \Rightarrow -BA = (-1)^k AB$

$\Rightarrow -AB = (-1)^k AB \Rightarrow AB = (-1)^{k+1} AB$

$\therefore$ k = odd number

3. (c) : Given, $y = 1 + x^2$ ...(i)

$\Rightarrow \frac{dy}{dx}=2x$

Hence $m=-\frac{1}{2x_1}$ (m = slope of normal at (x_1, y_1))

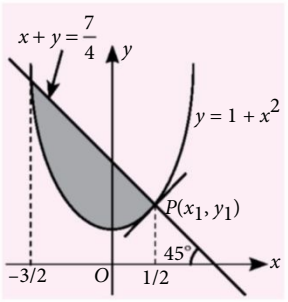
$\therefore x_1=\frac{-1}{2m}=\frac{1}{2}$; $y_1=1+\frac{1}{4m^2}=\frac{5}{4}$ [$\because m = -1$]

Equation of normal is $y-\frac{5}{4}=-\left(x-\frac{1}{2}\right)$

$\Rightarrow x+y=\frac{5}{4}+\frac{1}{2}=\frac{7}{4}$ ...(ii)

$A=\int_{x_1}^{x_2}\left(\left(\frac{7}{4}-x\right)-(1+x^2)\right)dx$

$=\int_{x_1}^{x_2}\left(\frac{3}{4}-x-x^2\right)dx$



Now, $1+x^2=-x+\frac{7}{4}$

$\Rightarrow 4x^2+4x-3=0 \Rightarrow x=-\frac{3}{2}$ and $x=\frac{1}{2}$

$\therefore A=\int_{-3/2}^{1/2}\left(\frac{3}{4}-x-x^2\right)dx=\left[\frac{3}{4}x-\frac{x^2}{2}-\frac{x^3}{3}\right]_{-3/2}^{1/2}$

$\Rightarrow A=\frac{4}{3}$ sq. units $\Rightarrow 3A = 4$.

4. (d) : Total number of cards = 52

Number of ace cards = 4

Number of non-ace cards = 48

$\therefore$ One ace card out of 4 can be selected in ${}^4C_1$ ways.

Remaining 4 cards out of 48 cards can be selected in ${}^{48}C_4$ ways.

$\therefore$ Required number of ways = ${}^4C_1 \times {}^{48}C_4$

$=\frac{4!}{3!\times 1!}\times\frac{48!}{4!\times 44!}=\frac{48\times 47\times 46\times 45}{6}=778320$

5. (a) : We have, $2^{n+10} = 2 \times 2^2 + 3 \times 2^3 + 4 \times 2^4 + ... + (n - 1) \times 2^{n-1} + n \times 2^n$ ...(i)

On multiplying both sides by 2, we get

$2 \times 2^{n+10} = 2 \times 2^3 + 3 \times 2^4 + ... + (n - 1)2^n + n \times 2^{n+1}$ ...(ii)

Subtracting (ii) from (i), we get

Admission Notice

U.P. State Entrance Examination (UPSEE-2019)

Dr. A.P.J. Abdul Kalam Technical University, Uttar Pradesh, Lucknow would conduct State Entrance Examination known as UPSEE-2019 for admission to Government/Government aided institutions and private unaided institutions affiliated to the Univtersity and some other State Universities of Uttar Pradesh for the session 2019-20. Applications are invited from eligible candidates for admission to :

A.	1st year of B. Tech / B.Tech (Biotech) / B. Tech (Ag) / B. Arch / B. Pharm / B. Des / BHMCT / BFAD / BFA / B. Voc
B.	2st year (Lateral Entry) of B. Tech / B. Pharm / MCA
C.	M. Tech / M. Tech Dual Degree / MBA / MBA integrated / MCA / MCA (Integrated) / M. Pharm / M. Arch / M. Des

Important Dates

Online Application Opens : 23rd January 2019, 14:00 Hrs.	**Date of Examination :** 21st April 2019
Online Application Closes : 15th March 2019, 17:00 Hrs.	**Results :** Last Week of May 2019

The information brochure containing detailed information about the eligibility and other requirements is available online on the Website https://www.upsee.nic.in. Candidates must read the information and check the details of the information brochure before filling online Application form.

$-2^{n+10} = 2 \times 2^2 + (2^3 + 2^4 + \ldots + 2^n) - n \times 2^{n+1}$

$\Rightarrow \quad -2^{n+10} = 8 + 8(2^{n-2} - 1) - n \times 2^{n+1}$

$\Rightarrow \quad -2^{n+10} = 2^{n+1} - n \times 2^{n+1}$

$\Rightarrow \quad -2^{10} = 2 - 2n \Rightarrow n = 513$

6. (c) : Here $f(x) = \begin{cases} 3 - x^2 & , \quad x \le 2 \\ \sqrt{a+14} - |x-48|, & x > 2 \end{cases}$

Since, $f(x)$ has a local maximum at $x = 2$.

$\therefore \quad \lim\limits_{h \to 0} f(2+h) \le f(2)$

$\Rightarrow \quad \lim\limits_{h \to 0} (\sqrt{a+14} - |2+h-48|) \le 3 - 2^2$

$\Rightarrow \quad \sqrt{a+14} \le 45 \Rightarrow a \le 2011$

7. (b) : Since, $\text{S.D.} = \sqrt{\dfrac{\sum x_i^2}{n} - \left(\dfrac{\sum x_i}{n}\right)^2}$

Now, $\dfrac{49}{4} = \dfrac{4+9+a^2+121}{4} - \left(\dfrac{16+a}{4}\right)^2$

$\Rightarrow \quad 49 \cdot 4 = 4(134 + a^2) - (256 + 32a + a^2)$

$\Rightarrow \quad 3a^2 - 32a + 84 = 0$

8. (a) : $\because$ Differentiation of $\sin x - \cos x$ repeat after fourth order.

$\therefore \dfrac{d^{16}y}{dx^{16}} = \sin x - \cos x \Rightarrow f(x) = \dfrac{d^{17}y}{dx^{17}} = \cos x + \sin x$

$\Rightarrow f\left(\dfrac{\pi}{4}\right) = \cos\dfrac{\pi}{4} + \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}} = \sqrt{2}$

$\therefore \left(\dfrac{1024}{\sqrt{2}}\right) f\left(\dfrac{\pi}{4}\right) = \left(\dfrac{1024}{\sqrt{2}}\right)(\sqrt{2}) = 1024$

9. (a, b, c) : If we put $x = \tan\theta$, the given equality becomes $\tan^{-1} y = 4\theta$.

$\Rightarrow \quad y = \tan 4\theta = \dfrac{2\tan 2\theta}{1 - \tan^2 2\theta} = \dfrac{2\left[\dfrac{2\tan\theta}{1-\tan^2\theta}\right]}{1 - \left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)^2}$

$= \dfrac{2 \times 2x(1-x^2)}{(1-x^2)^2 - 4x^2} = \dfrac{4x(1-x^2)}{1 - 6x^2 + x^4}$

Now, y is infinite if $x^4 - 6x^2 + 1 = 0$

$\Rightarrow \quad x^2 = \dfrac{6 \pm \sqrt{36-4}}{2} = 3 \pm 2\sqrt{2}$

10. (b) : Since $\dfrac{a}{b} = \dfrac{2+\sqrt{3}}{1} \Rightarrow \angle A > \angle B$

Also, $\dfrac{a}{b} = \dfrac{\sin 105^\circ}{\sin 15^\circ} = 2 + \sqrt{3}$, which is true.

11. (c) : Here $f(x) = x^4 + ax^3 + \dfrac{3x^2}{2} + 1$

$\Rightarrow f'(x) = 4x^3 + 3ax^2 + 3x \Rightarrow f''(x) = 12x^2 + 6ax + 3$

Now, $f(x)$ will be concave upward along the entire real line if $f''(x) \ge 0 \ \forall \ x \in R$

$\Rightarrow \quad 12x^2 + 6ax + 3 \ge 0 \ \forall \ x \in R \quad \Rightarrow \quad D \le 0$

$\Rightarrow \quad 36a^2 - 144 \le 0 \quad \Rightarrow \quad a^2 - 4 \le 0 \quad \Rightarrow \quad a \in [-2, 2]$

12. (a, b, c) : $I = \dfrac{1}{\sin(a-b)} \int \dfrac{\sin(x-b-(x-a))\,dx}{\cos(x-a)\cos(x-b)}$

$= \dfrac{1}{\sin(a-b)} \int (\tan(x-b) - \tan(x-a))\,dx$

13. (b) : $f(x) = \cos(2\pi[x] + \pi|x|) = \cos(\pi|x|) = \cos(\pi x)$

Clearly $f(x)$ is an even function with range $[-1,1]$.

14. (a, b) : $\overrightarrow{BC} = (a\hat{i} - 3\hat{j} + \hat{k}) - (\hat{i} - 3\hat{j} - 5\hat{k})$

$= (a-1)\hat{i} + 6\hat{k}$

$\overrightarrow{CA} = (2\hat{i} - \hat{j} + \hat{k}) - (a\hat{i} - 3\hat{j} + \hat{k}) = (2-a)\hat{i} + 2\hat{j}$

$\overrightarrow{BC} \cdot \overrightarrow{CA} = 0 \Rightarrow (a-1)(2-a) = 0$

$\therefore \quad a = 1, 2.$

15. (d) : $\lim\limits_{x \to 0} \dfrac{e^{2x} - (1+4x)^{1/2}}{\log(1-x^2)} = \lim\limits_{x \to 0} \dfrac{e^{2x} - (1+4x)^{1/2}}{\dfrac{\log(1-x^2)}{-x^2}(-x^2)}$

$= \lim\limits_{x \to 0} \dfrac{(1+4x)^{1/2} - e^{2x}}{x^2}$

$= \lim\limits_{x \to 0} \dfrac{\left(1 + \dfrac{1}{2}4x + \dfrac{1}{2}\cdot\left(\dfrac{1}{2}-1\right)\dfrac{1}{2!}16x^2 + \ldots\right) - \left(1 + \dfrac{2x}{1!} + \dfrac{4x^2}{2!} + \ldots\right)}{x^2}$

$= -4$

16. (b, c, d) : $\begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix}$

Applying $C_3 \to C_3 - \alpha\, C_1 - C_2$, we get

$\begin{vmatrix} a & b & 0 \\ b & c & 0 \\ a\alpha + b & b\alpha + c & -(a\alpha^2 + 2b\alpha + c) \end{vmatrix}$

$= (b^2 - ca)(a\alpha^2 + 2b\alpha + c) = 0$

if a, b, c are in G.P. or α is a root of $ax^2 + 2bx + c = 0$.

MPP CLASS XI ANSWER KEY

1.	(b)	**2.**	(a)	**3.**	(c)	**4.**	(b)	**5.**	(d)
6.	(c)	**7.**	(a,d)	**8.**	(a,b)	**9.**	(a,b,c)	**10.**	(a,b,c)
11.	(a)	**12.**	(a,b,d)	**13.**	(a,b,c)	**14.**	(b)	**15.**	(d)
16.	(b)	**17.**	(100)	**18.**	(786)	**19.**	(36)	**20.**	(6)

17. (a, b, d) : Let d be the common difference and $a_m = 2$, $a_n = 31$, then $(n - m)d = 29 \Rightarrow d = \frac{29}{n-m}$ which is rational.

Also $a_r - a_m = (r - m)d \quad \Rightarrow \quad a_r$ is rational.

As each term is rational, sum to any finite number of terms must be rational.

18. (b, c) : Here, $f(x) = x^3 - x^2 + 100x + 2002$

$\Rightarrow \quad f'(x) = 3x^2 - 2x + 100 > 0 \ \forall \ x \in R$

$\therefore \quad f(x)$ is strictly increasing.

Therefore, $f\left(\frac{1}{2000}\right) > f\left(\frac{1}{2001}\right)$

Also, $f(x - 1) > f(x - 2)$ as $x - 1 > x - 2$ for $\forall\ x$.

19. (a, c) : $\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$

$\Rightarrow \quad p(rq - p^2) - q(q^2 - rp) + r(pq - r^2) = 0$

or $\quad p^3 + q^3 + r^3 - 3pqr = 0$

Now, $p^3+q^3+r^3-3pqr=(p+q+r)(p^2+q^2+r^2-pq-qr-rp)$

$\therefore \quad p + q + r = 0$

But $p^2 + q^2 + r^2 - pq - qr - rp = 0$ only if $p = q = r$ in which case all the three lines are same, $x + y + 1 = 0$.

20. (a, b) : The three points lie in the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ (intercept form) and area of $\Delta ABC=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

$=\frac{1}{2}|(-a\hat{i}+b\hat{j})\times(-a\hat{i}+c\hat{k})|$

21. (d) : Since, roots of $f(x) = 0$ are equal in magnitude and opposite in sign.

$\therefore$ Sum of the roots = 0 and product of the roots < 0

Sum of the roots = 0 $\Rightarrow a = 0$ ...(i)

Product of the roots < 0 $\quad \Rightarrow \quad 5a^2 - 6a < 0$

$\Rightarrow \quad a \in \left(0, \frac{6}{5}\right)$ ...(ii)

From (i) and (ii), $a = \phi$

22. (b) : If one root of $f(x) = 0$ is zero, then product of the roots is 0

$\Rightarrow \quad 5a^2 - 6a = 0 \quad \Rightarrow \quad a = 0, \frac{6}{5}$

Hence, $a = \frac{6}{5}$ $\quad (\because \ a \neq 0$ as sum of the roots is non zero.)

23. (b) : Range of $f(x)$ is $[-5, \infty)$ for every real x or range of $f(x) + 5$ is $[0, \infty)$ for every real x.

Now $f(x) + 5 = x^2 - 4ax + 5a^2 - 6a + 5$

For range to be $[0, \infty)$, $D = 0$

$\Rightarrow \quad 16a^2 - 4(5a^2 - 6a + 5) = 0$

$\Rightarrow \quad a^2 - 6a + 5 = 0 \Rightarrow a = 5$ or 1

24. (d) : $\lim\limits_{x\to -1}\frac{f(x)}{(x+1)^3} = 1$ [Given]

$\Rightarrow \quad f(-1) = f'(-1) = f''(-1) = 0, f'''(-1) = 6$

$\therefore \quad f(x) = A(x + 1)^4 + (x + 1)^3$

$\Rightarrow \quad f'''(x) = 24A(x + 1) + 6$

$f'''(0) = 0 \Rightarrow 24A + 6 = 0 \Rightarrow A = -\frac{1}{4}$

$\therefore \quad f(x) = (x + 1)^3 - \frac{1}{4}(x + 1)^4$

$\Rightarrow \quad f'(x) = 3(x + 1)^2 - (x + 1)^3$

$\Rightarrow \quad f''(x) = 6(x + 1) - 3(x + 1)^2$

$f'(x) = 0 \Rightarrow (x + 1)^2(2 - x) = 0$

$\Rightarrow \quad x = -1, x = 2$ and $f''(2) = -9$

So, $x = 2$ is a point of maxima.

Thus, the maximum value is $f(2) = 27 - \frac{81}{4} = \frac{27}{4}$

Is appearing for JEE Main second time a good idea?

NTA conducted JEE Main 1 from 9th to 12th January. For the aspirants now is the second chance to buckle themselves up to face the fierce competition in the race to achieve their goal.

The second session of JEE Main is in April. In order to appear in JEE Main April 2019 Examination the candidates are required to apply only online between 8th February 2019 to 07th March 2019. The fees can be paid online up to 8th March 2019.

There is no harm in attempting the exam second time as this is the time to improve your score as if the candidate appears in both the January and April sessions, then the better of the two scores will be considered.

For JEE Main January 2019:

Number of candidates registered (Paper-I : B.E./B.Tech.)	9,29,198
Number of candidates appeared (Paper-I : B.E./B.Tech.)	8,74,469

While the percentile score has made 13 students get a perfect score, there would be only one who would attain all India rank (AIR) 1. This has led to a more cut-throat competition, with even the 100 percentile scorers attempting for a better rank. Despite scoring decent percentile, many candidates are planning to re-appear for the JEE Main.

Cracking JEE Main and later JEE Advanced will ensure a seat in architecture and engineering graduate courses in country's top universities, but it is better for those who have secured below expectations in January exam to focus on board exams first and start their preparation for JEE 2020 immediately after the board exam as the eligibility criteria sates that the candidate should have secured at least 75% marks in the 12th class examination, or be in the top 20 percentile in the 12th class examination conducted by the respective Boards. For SC/ST candidates the qualifying marks would be 65% in the 12th class examination.

25. (b) : $f(0) = 1 - \frac{1}{4} = \frac{3}{4}$, $f'(0) = 2$

Subnormal at $x = 0$ is $f(0)\, f'(0) = \frac{3}{4} \times 2 = \frac{3}{2}$

26. (c) : $f''(x) = 3(x + 1)\,(2 - x - 1) = 3(x + 1)(1 - x)$

$f''(x) = 0 \Rightarrow x = 1, -1$

27. (b) : The vector perpendicular to L_1 and L_2 is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -\hat{i} - 7\hat{j} + 5\hat{k}.$$

$\therefore$ Unit vector $(\hat{n})$ is $\frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$.

28. (d) : The points on L_1 and L_2 are $A(-1, -2, -1)$ and $C(2, -2, 3)$

The shortest distance is

$$\overrightarrow{AC} \times \hat{n} = (3\hat{i} + 4\hat{k}) \cdot \frac{(-\hat{i} - 7\hat{j} + 5\hat{k})}{5\sqrt{3}} = \frac{17}{5\sqrt{3}}.$$

29. (c) : The *d.r's* of the normal are $-1, -7, 5$

$\therefore$ The plane is $-(x + 1) - 7(y + 2) + 5(z + 1) = 0$

$\Rightarrow x + 7y - 5z + 10 = 0$

The distance of $(1, 1, 1)$ is $13/\sqrt{75} = 13/5\sqrt{3}$.

30. (c) : **(A)** $\because {}^nC_0 + {}^nC_1 + {}^nC_2 = 46$

$$\Rightarrow 1 + n + \frac{n(n-1)}{2} = 46$$

$\Rightarrow 2 + 2n + n^2 - n = 92 \Rightarrow n^2 + n - 90 = 0$

$\Rightarrow (n - 9)\,(n + 10) = 0 \Rightarrow n = 9, n \neq -10$

$$\therefore \left(x^2 + \frac{1}{x}\right)^n = \left(x^2 + \frac{1}{x}\right)^9$$

$$\therefore T_{r+1} = {}^9C_r (x^2)^{9-r} \cdot \left(\frac{1}{x}\right)^r = {}^9C_r \cdot x^{18-3r}$$

Now, $18 - 3r = 0 \Rightarrow r = 6$

$$\Rightarrow T_{r+1} = {}^9C_6 = {}^9C_3 = \frac{9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3} = 84$$

(B) $T_{r+1} = {}^nC_r \left(\frac{5}{x^2}\right)^{n-r} \cdot (x^4)^r$

$= {}^nC_r \cdot (5)^{n-r} \cdot x^{-2n+2r+4r} = {}^nC_r \cdot (5)^{n-r} \cdot x^{6r-2n}$

$\because 6r - 2n = 0 \Rightarrow r = n/3 \quad \therefore \quad n = 3, 6, 9, \ldots..$

(C) $\because T_{r+1} = {}^nC_r (x^3)^{n-r} \cdot \left(-\frac{1}{x^2}\right)^r = {}^nC_r \cdot (-1)^r \cdot x^{3n-5r}$

For coefficient of x^5, we get $3n - 5r = 5$

$$\Rightarrow r = \frac{3n-5}{5} = p(\text{say})$$

For coefficient of x^{10}, we get $3n - 5r = 10$

$$\Rightarrow r = \frac{3n-10}{5} = q(\text{say})$$

$\because p - q = 1$ (using values of r) ...(i)

Also, ${}^nC_p(-1)^p + {}^nC_q(-1)^q = 0$

$\Rightarrow {}^nC_p(-1)^p + {}^nC_{p-1}(-1)^{p-1} = 0$ (From (i))

$\Rightarrow (-1)^p\{{}^nC_p - {}^nC_{p-1}\} = 0 \Rightarrow {}^nC_p - {}^nC_{p-1} = 0$

$\Rightarrow {}^nC_p = {}^nC_{p-1} \Rightarrow n = p + p - 1 \Rightarrow p = \frac{n+1}{2}$

$\therefore r = p \Rightarrow \frac{3n-5}{5} = \frac{n+1}{2}$

$\Rightarrow 6n - 10 = 5n + 5 \Rightarrow n = 15$

(D) We have, $(1 + x + x^2)^4$

$= (1 + x\,(1 + x))^4 = 1 + {}^4C_1\, x(1 + x) + {}^4C_2\, x^2\,(1 + x)^2 + {}^4C_3\, x^3\,(1 + x)^3 + {}^4C_4\, x^4\,(1 + x)^4$

$= 1 + 4x\,(1 + x) + 6x^2\,(1 + x^2 + 2x) + 4x^3\,(1 + x^3 + 3x + 3x^2) + x^4\,(x^4 + 4x^3 + 6x^2 + 4x + 1)$

$\therefore$ Coefficient of $x^4 = 6 + 12 + 1 = 19$

31. (d) : **(A)** $C_1 : x^2 + y^2 - 2a(x + y) + a^2 = 0$,

Centre : (a, a), radius : a,

$C_2 : x^2 + y^2 - 2b(x + y) + b^2 = 0$

Centre : (b, b), radius b

Since C_1 and C_2 touch each other

$$\Rightarrow \sqrt{2}(b - a) = a + b \Rightarrow \frac{b}{a} = (\sqrt{2} + 1)^2 = 3 + 2\sqrt{2}$$

(B) C_1 and C_2 intersect orthogonally

$$\Rightarrow 2(b - a)^2 = a^2 + b^2 \Rightarrow \frac{b}{a} = 2 + \sqrt{3}$$

(C) The common chord is

$2(b - a)(x + y) = b^2 - a^2$.

It passes through $(a, a) \Rightarrow b/a = 3$.

(D) C_2 passes through (a, a)

$$\Rightarrow 2a^2 - 4ab + b^2 = 0 \Rightarrow \frac{b}{a} = 2 + \sqrt{2}.$$

32. (a) : **(A)** The value of $\{x\} = x - 2$ for x close to 2 and $x > 2$

$$L = \lim_{x \to 2^+} \frac{(x-2)\sin(x-2)}{(x-2)^2} = \lim_{x \to 2^+} \frac{\sin(x-2)}{x-2} = 1.$$

(B) $\lim_{x \to 2} [2 - x] + [x - 2] - x$

$= \lim_{x \to 2} 2 + [-x] + [x] - 2 - x = \lim_{x \to 2} [-x] + [x] - x$

R.H.L. $= \lim_{x \to 2^+} [-x] + [x] - 2 = -3 + 2 - 2 = -3$

L.H.L. $= \lim_{x \to 2^-} [-x] + [x] - 2 = -2 + 1 - 2 = -3$

$\therefore \quad L = -3.$

(C) $\{2 - x\} + \{x - 2\} = 2 - x - [2 - x] + x - 2 - [x - 2]$

$= -[2 - x] - [x - 2] = -2 - [-x] - [x] + 2 = -[-x] - [x]$

R.H.L.$= \lim_{x \to 2^+} -[-x] - [x] = 3 - 2 = 1$

L.H.L.$= \lim_{x \to 2^-} -[-x] - [x] = 2 - 1 = 1 \quad \therefore \quad$ Limit $= 1$.

(D) R.H.L. $= \lim\limits_{x\to 0^+} \dfrac{e^{\frac{1}{x}} - e^{\frac{-1}{x}}}{e^{\frac{1}{x}} + e^{\frac{-1}{x}}} = \lim\limits_{x\to 0} \dfrac{1-e^{\frac{-2}{x}}}{1+e^{\frac{-2}{x}}} = \dfrac{1-0}{1+0} = 1$

33. (4) : Putting $x = 1$ and $y = 1$, we get

$f^2(1) - f(1) - 6 = 0 \Rightarrow f(1) = 3$ or $f(1) = -2$

Now, putting $y = 1$, we get

$$f(x)\cdot f(1) = f(x) + 2\left(\frac{1}{x} + 2\right) = f(x) + 2\left(\frac{2x+1}{x}\right)$$

$$\Rightarrow \quad f(x)[f(1)-1] = \frac{2(2x+1)}{x} \quad \Rightarrow \quad f(x) = \frac{2(2x+1)}{x[f(1)-1]}$$

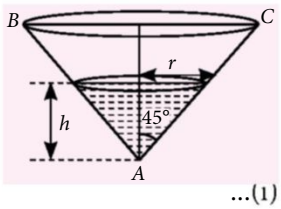
For $f(1) = 3$, $f(x) = \dfrac{(2x+1)}{x}$ ...(i)

and for $f(1) = -2$, $f(x) = \dfrac{2(2x+1)}{-3x}$ ...(ii)

$$\Rightarrow \quad f\left(\frac{1}{2}\right) = 4$$

34. (5) : We have,

$$\frac{dV}{dt} = 2 \text{ or } \frac{d}{dt}\left(\frac{1}{3}\pi r^3\right) = 2$$



[$\because r = h$ as $\theta = 45°$]

$$\Rightarrow \pi r^2 \frac{dr}{dt} = 2 \Rightarrow \frac{dr}{dt} = \frac{2}{\pi r^2} \quad ...(1)$$

Now, perimeter $= 2\pi r = p$ (say)

$$\therefore \quad \frac{d}{dt}(2\pi r) = 2\pi\frac{dr}{dt} = 2\pi\left(\frac{2}{\pi r^2}\right) = \frac{4}{r^2} \quad ...(ii)\,[\text{Using (i)}]$$

When $h = 2$ m, $r = 2$ m

Hence, $\dfrac{dp}{dt} = \dfrac{4}{4} = 1$ m/s.

35. (8) : Let $\alpha = \sin^{-1}\dfrac{\sqrt{5}}{3}$, then $\cos^{-1}\dfrac{\sqrt{5}}{3} = \dfrac{\pi}{2} - \alpha$

Hence the expression $= \sin\left\{2\left(\alpha - \dfrac{\pi}{2} + \alpha\right)\right\}$

$= \sin(4\alpha - \pi) = -\sin 4\alpha = -4\sin\alpha\cos\alpha(1 - 2\sin^2\alpha)$

$$= (-4)\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)\left(\frac{-1}{9}\right) = \frac{8\sqrt{5}}{81}$$

36. (1) : As $f(-1) = f(1)$ and Roll's theorem is not applicable, then it implies $f(x)$ is either discontinuous or $f'(x)$does not exist at atleast one point in $(-1, 1)$. $g(x) = 0$ at atleast one value of x in $(-1, 1)$. Thus $k = 1$

37. (104) : The required number of points $= {}^8C_2 \times 1 + {}^4C_2 \times 2 + ({}^8C_1 \times {}^4C_1) \times 2 = 28 + 12 + 32 \times 2 = 104$.

38. (8) : $I = \int (x^9 + x^6 + x^3)(2x^6 + 3x^3 + 6)^{1/3}\, dx$

$$= \int x^2(x^6 + x^3 + 1)(2x^9 + 3x^6 + 6x^3)^{1/3} dx$$

Put $2x^9 + 3x^6 + 6x^3 = t, 18(x^8 + x^5 + x^2)dx = dt$

$\Rightarrow \quad 18x^2(x^6 + x^3 + 1)dx = dt$

$\therefore \quad I = \dfrac{1}{24}(2x^9 + 3x^6 + 6x^3)^{4/3} + K \quad \therefore \quad \dfrac{AB}{4} = 8$

39. (15) : $np = 2$ ...(i), $npq = 1$ ...(ii)

$\Rightarrow \quad q = \dfrac{1}{2}, \; p = \dfrac{1}{2}, \; n = 4$ (from (i) and (ii))

$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0)$

$$= 1 - {}^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4 = 1 - \frac{1}{16} = \frac{15}{16}$$

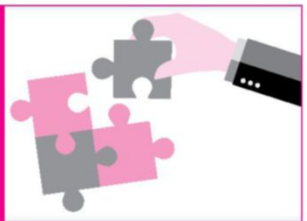
40. (9) : $\vec{r}\times\vec{b} = \vec{c}\times\vec{b} \Rightarrow (\vec{r} - \vec{c})\times\vec{b} = \vec{0}$

$\Rightarrow \vec{r} = \vec{c} + \lambda\vec{b}$ ...(i)

$$\Rightarrow \quad \vec{r}\cdot\vec{a} = \vec{a}\cdot\vec{c} + \lambda\vec{a}\cdot\vec{b} \Rightarrow \vec{r} = \vec{c} - \frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\vec{b}$$

$$\Rightarrow \quad \vec{r}\cdot\vec{b} = \vec{b}\cdot\vec{c} - \frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\left(|\vec{b}|^2\right) = 1 - \frac{-4}{1}(2) = 9$$

PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics.

In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is multiple of its numbers. For example, if that value is 3 for a two-box cluster, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

450×		2+		6+	
		3+	1–		3–
10+	1			12+	
		16+			13+
2+	3+				
	1–		5–		

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

JEE 2019 ADVANCED

Exam on 19th May

PRACTICE PAPER

*ALOK KUMAR

Single Option Correct Type

1. If $f(x)=\left(\sum_{r=0}^{n} a_r\,|x-1|^r\right)$, where a_r's are positive real constants, then $f(x)$ is

(a) not continuous at $x = 1$

(b) continuous everywhere but not differentiable at $x = 0$

(c) continuous everywhere but not differentiable at $x = 1$

(d) differentiable everywhere

2. If $x + y + z = \pi$, $\tan x \tan z = 2$ and $\tan y \tan z = 18$, then $\tan^2 z =$

(a) 15 (b) 16 (c) 19 (d) 20

3. 8 players $P_1, P_2, \ldots\ldots, P_8$ of equal strength play in a knockout tournament. Assuming that players in each round are paired randomly, the probability that the player P_1 loses to the eventual winner is

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{5}{8}$ (d) $\frac{7}{8}$

4. If z_1 and z_2 are two complex numbers such that $|z_1| = 2$ and $(1-i)z_2 + (1+i)\bar{z}_2 = 8\sqrt{2}$, then the minimum value of $|z_1 - z_2|$ is

(a) 1 (b) 2 (c) 3 (d) 4

5. Let $f(x)$ be a continuous function which takes positive values for $x \geq 0$ and $\int_0^x f(t)\,dt = x\sqrt{f(x)}$ with $f(1)=\frac{1}{2}$. Then the value of $f(\sqrt{2}+1)$ is

(a) 1 (b) 2 (c) 4 (d) $\frac{1}{4}$

6. Let L_1 and L_2 be the lines $\vec{r} = (2\hat{i}+\hat{j}-\hat{k}) + \lambda(\hat{i}+2\hat{k})$ and $\vec{r} = (3\hat{i}+\hat{j}) + \mu(\hat{i}+\hat{j}-\hat{k})$. If the plane π which contains L_1 and parallel to L_2 meets the coordinate axes at A, B and C respectively, then the volume of the tetrahedron $OABC$ is

(a) $\frac{4}{9}$ (b) $\frac{4}{3}$ (c) $\frac{2}{9}$ (d) $\frac{2}{3}$

7. The area bounded by the curve $y = f(x)$, the x–axis and the line $y = 1$, where $f(x)=1+\frac{1}{x}\int_1^x f(t)\,dt$, $(x \neq 0)$ is

(a) $2\left(1-\frac{1}{e}\right)$ (b) $2\left(1+\frac{1}{e}\right)$

(c) $\left(1+\frac{1}{e}\right)$ (d) $\left(1-\frac{1}{e}\right)$

8. The number of points (b, c) lying on the circle $x^2 + (y - 3)^2 = 8$ such that the equation $x^2 + bx + c = 0$ has real roots is

(a) 1 (b) 2 (c) 3 (d) 4

One or More Than One Option(s) Correct Type

9. Let A, B, C be three angles such that $A=\frac{\pi}{4}$ and $\tan B \tan C = p$. The set of all possible values of p such that A, B, C are the angles of a triangle contains

(a) $(-\infty, 0)$ (b) $(0, 1)$

(c) $(1, 3+2\sqrt{2})$ (d) $\left[3+2\sqrt{2}, \infty\right)$

10. Consider the function $f(x) = \sin^5 x + \cos^5 x - 1$, $x \in \left[0, \frac{\pi}{2}\right]$. Which of the following is/are correct?

*Alok Kumar, a B Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

(a) f is strictly decreasing in $\left(0, \frac{\pi}{4}\right)$

(b) f is strictly increasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(c) There exists a number 'c' in $\left(0, \frac{\pi}{2}\right)$ such that $f'(c) = 0$

(d) The equation $f(x) = 0$ has only two roots in $\left[0, \frac{\pi}{2}\right]$

11. Let ABC be a triangle with $\angle BAC = 120°$ and $AB \cdot AC = 1$. Also, let AD be the length of the angle bisector of $\angle A$ of the triangle. Then

(a) Minimum value of AD is $\frac{1}{2}$

(b) Maximum value of AD is $\frac{1}{2}$

(c) AD is minimum when ΔABC is isosceles

(d) AD is maximum when ΔABC is isosceles

Comprehension Type

Paragraph for Q. No. 12 to 14

Let $f(x) = \lim\limits_{n\to\infty}\left(\cos\sqrt{\frac{x}{n}}\right)^n$, $g(x) = \lim\limits_{n\to\infty}\left(1 - x + x\sqrt[n]{e}\right)^n$.

Now, consider the function $y = h(x)$, where $h(x) = \tan^{-1}(g^{-1}f^{-1}(x))$.

12. $\lim\limits_{x\to 0^+}\frac{\ln(f(x))}{\ln(g(x))}$ is equal to

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 0 (d) 1

13. Domain of the function $y = h(x)$ is

(a) $(0, \infty)$ (b) R (c) $(0, 1)$ (d) $[0, 1]$

14. Range of the function $y = h(x)$ is

(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left(-\frac{\pi}{2}, 0\right)$

(c) R (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Paragraph for Q. No. 15 to 17

$y = f(x)$ is a parabola of the form $y = x^2 + ax + 1$, its tangent at the point of its intersection with y-axis also touches the circle $x^2 + y^2 = r^2$. It is known that no point of parabola is below x-axis.

15. The radius of circle when 'a' attains its maximum value is

(a) $\frac{1}{\sqrt{10}}$ (b) $\frac{1}{\sqrt{5}}$ (c) 1 (d) $\sqrt{5}$

16. The slope of the tangent when radius of the circle is maximum, is

(a) 0 (b) 1 (c) -1 (d) not defined

17. The minimum area bounded by the tangent and the coordinate axes is

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

Matrix-Match Type

18. Match the following :

Column -I		Column -II	
(A)	If $f(x) = \begin{cases} 4x - x^3 + \log_e(a^2 - 3a + 3), & \text{if } 0 \le x < 3 \\ x - 18, & \text{if } x \ge 3 \end{cases}$ then complete set of value of a such that $f(x)$ has a local maxima at $x = 3$ is	(p)	$(-\infty, -1] \cup [1, \infty)$
(B)	If the equation $x + \cos x = a$ has exactly a positive root, then complete set of values of a is	(q)	$[1, 2]$
(C)	If $f(x) = \cos x + a^2x + b$ is an increasing function for all values of x then the complete set of values of a is	(r)	$(-20, -16)$
(D)	If the function $f(x) = x^3 - 9x^2 + 24x + a$ has 3 real and distinct roots α, β and γ then the possible values of a are	(s)	$(1, \infty)$

19. Match the following :

Column -I		Column -II	
(A)	If $\log_3(a+b)+\log_3(c+d)\geq 4$ then the minimum value of $a+b+c+d$ is	(p)	18
(B)	The number of distinct terms in the expansion of $\left(x+y+\frac{1}{x}+\frac{1}{y}\right)^{14}$ is	(q)	225
(C)	The remainder when $(23)^{86}$ is divided by 100 is	(r)	89
(D)	If the third term in the expansion of $\left(\frac{1}{x}+x^{\log_{10}x}\right)^5$ is 1000 and $x>1$, then the value of x is	(s)	100

Numerical Answer Type

20. Let x, y, z, t be real numbers such that $x^2+y^2=9$, $z^2+t^2=4$, and $xt-yz=6$. Then greatest value of $P=xz$ is

21. A cricket player played $n(n>1)$ matches during his career and made a total of $\frac{(n+1)(2^{n+1}-n-2)}{4}$ runs. If the player made $k\cdot 2^{n-k+1}$ runs in the k^{th} match $(1\leq k\leq n)$, the value of 'n' is equal to

22. Let G_1, G_2 and G_3 be the centroids of the triangular faces OBC, OCA and OAB of a tetrahedron $OABC$. If V_1 denotes the volume of tetrahedron $OABC$ and V_2 that of the parallelepiped with OG_1, OG_2 and OG_3 as three concurrent edges, then the value of $4V_1/V_2$ is (where O is the origin)

SOLUTIONS

1. (c) : $|x-1|$, $|x-1|^2$ etc are all continuous everywhere and the algebraic sum of continuous functions is also continuous. $|x-1|$, $|x-1|^3$ etc, are not differentiable at $x=1$ whereas $|x-1|^2$, $|x-1|^4$ etc. are all differentiable at $x=1$.

2. (b) : Given $x+y+z=\pi$

$\Rightarrow$ $\tan x+\tan z=\tan y$

$\Rightarrow$ $(\tan x+\tan z)\tan z=\tan y\tan z=18$

$\Rightarrow$ $\tan^2 z=18-\tan x\tan z=18-2=16$

3. (b) : If E_1, E_2, E_3 are the events of P_1 losing to the champion in the 1st, 2nd and 3rd rounds, then the required probability

$$P(E_1)+P(E_2)+P(E_3)=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$$

4. (b) : $\text{Re}(1-i)z_2=4\sqrt{2} \Rightarrow |(1-i)z_2|\geq 4\sqrt{2}$

$\Rightarrow$ $|z_2|\geq 4 \Rightarrow$ z_2 lies either on the circumference or outside the circle with centre at origin and radius 4.

$\therefore$ $|z_1-z_2|\geq 2$

5. (d) : The given relation reduces to the differential equation

$$x\frac{dy}{dx}+2y=2y^{3/2} \Rightarrow \frac{dz}{dx}-\frac{z}{x}=\frac{1}{x}, \text{ where } z=\frac{-1}{\sqrt{y}}$$

$\therefore$ The required solution is $\frac{z}{x}=\frac{-1}{x}+c$,

where $c=1-\sqrt{2}$ $\left(\text{by using } f(1)=\frac{1}{2}\right)$

$$\therefore \frac{1}{\sqrt{y}}=(\sqrt{2}-1)x+1 \Rightarrow y(\sqrt{2}+1)=\frac{1}{4}$$

6. (c) : Equation of the plane containing L_1 and parallel to L_2 is $\begin{vmatrix} x-2 & y-1 & z+1 \\ 1 & 0 & 2 \\ 1 & 1 & -1 \end{vmatrix}=0$

i.e., $2x-3y-z-2=0$

This plane meets the axes at $A(1, 0, 0)$, $B(0, -2/3, 0)$ and $C(0, 0, -2)$

$\therefore$ Volume of tetrahedron $OABC=\frac{1}{6}|abc|=\frac{2}{9}$

7. (a) : $f(x)=1+\ln|x|$

$$\text{Area}=2\int_0^1 e^{y-1}dy=2\left(1-\frac{1}{e}\right)$$

8. (b) : $b^2\geq 4c \Leftrightarrow 8-(c-3)^2\geq 4c$

$\Leftrightarrow$ $c^2-2c+1\leq 0 \Leftrightarrow c=1 \therefore b=\pm 2$

9. (a, d) : $\tan B\tan C=p$

$$\Leftrightarrow \frac{\cos(B-C)}{\cos(B+C)}=\frac{p+1}{1-p} \Leftrightarrow \cos(B-C)=\frac{p+1}{\sqrt{2}(p-1)}$$

$$\therefore 0\leq|B-C|<\frac{3\pi}{4} \Leftrightarrow \frac{-1}{\sqrt{2}}<\frac{p+1}{\sqrt{2}(p-1)}\leq 1$$

$$\Leftrightarrow \frac{p+1}{p-1}+1=\frac{2p}{p-1}>0 \text{ and } \frac{(p+1)-\sqrt{2}(p-1)}{(p-1)}\leq 0$$

$$\Leftrightarrow p(p-1)>0 \text{ and } [(\sqrt{2}+1)-(\sqrt{2}-1)p](p-1)\leq 0$$

$\Leftrightarrow \quad p \notin [0,1]$ and $[(\sqrt{2}+1)^2 - p](p-1) \le 0$

$\Leftrightarrow \quad p \notin [0,1]$ and $p \notin (1, 3+2\sqrt{2})$

$\Leftrightarrow \quad p \notin (0, 3+2\sqrt{2}) \Leftrightarrow p < 0$ or $p \ge 3+2\sqrt{2}$

10. (a, b, c, d) : $f'(x) = (5\sin x \cos x)(\sin^3 x - \cos^3 x)$

Now, $f'(x) < 0$ for $x \in \left(0, \frac{\pi}{4}\right)$ and $f'(x) > 0$ for $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Since, $f(0) = 0 = f\left(\frac{\pi}{2}\right)$, so applying Rolles theorem to f on $(0, \pi/2)$ we observe that $f'(c) = 0$ for atleast one c in $(0, \pi/2)$. Also, $1 = \sin^5 x + \cos^5 x \le \sin^2 x + \cos^2 x = 1$ for $x \in \left[0, \frac{\pi}{2}\right]$ holds only if $\sin^5 x = \sin^2 x$ and $\cos^5 x = \cos^2 x$ *i.e.*, only if $x \in \left[0, \frac{\pi}{2}\right]$.

11. (b, d) : Let $AB = x$. Then $BC^2 = x^2 + \frac{1}{x^2} + 1$

and $\cos B = \frac{2x^2+1}{2\sqrt{x^4+x^2+1}}$ *i.e.*, $\tan B = \frac{\sqrt{3}}{2x^2+1}$

Also, $\frac{AD}{\sin B} = \frac{x}{\sin(B+60°)} \Rightarrow AD = \frac{x}{\frac{1}{2} + \frac{\sqrt{3}}{2}\cot B} = \frac{x}{x^2+1}$

$\therefore \quad AD = \frac{1}{x + \frac{1}{x}} \le \frac{1}{2}$, with equality iff $AB = AC = 1$

(12-14) :

12. (b) 13. (c) 14. (d)

$$f(x) = \lim_{n\to\infty}\left(\cos\sqrt{\frac{x}{n}}\right)^n = \lim_{n\to\infty}\left(1 + \left(\cos\sqrt{\frac{x}{n}} - 1\right)\right)^n$$

$$= e^{\lim_{n\to\infty}\left(\cos\sqrt{\frac{x}{n}} - 1\right)n} = e^{-\lim_{n\to\infty} 2\sin^2\left(\frac{1}{2}\sqrt{\frac{x}{n}}\right)n}$$

$$= e^{-2\lim_{n\to\infty}\frac{\left(\frac{1}{2}\sqrt{\frac{x}{n}}\right)^2}{\frac{1}{n}}} = e^{-2\lim_{n\to\infty}\frac{1}{4}\frac{\frac{x}{n}}{\frac{1}{n}}} = e^{-x/2}$$

$y = f(x) = e^{-x/2}$, $x \ge 0$ range $= (0, 1]$

$$g(x) = \lim_{n\to\infty}(1 - x + x\sqrt[n]{e})^n = e^{\lim_{n\to\infty} x\frac{(e^{1/n}-1)}{1/n}} = e^x \quad \forall\ x \in R$$

$h(x) = \tan^{-1}(g^{-1}(f^{-1}(x)))$

Now, $f^{-1}(x) = 2\ln\frac{1}{x}$ $0 < x < 1$; $g^{-1}(x) = \ln x$

$\therefore \quad g^{-1}\left(2\ln\frac{1}{x}\right) = \ln\left(2\ln\left(\frac{1}{x}\right)\right)$ for $0 < x < 1$.

$\therefore \quad h(x) = \tan^{-1}\left(\ln\left(\ln\frac{1}{x^2}\right)\right)$ for $0 < x < 1$

Now, $\lim_{x\to 0^+}\frac{\ln f(x)}{\ln g(x)} = \lim_{x\to 0}\frac{-x/2}{x} = -\frac{1}{2}$

Now, $h(x) = \tan^{-1}(\ln(\ln 1/x^2))$, $0 < x < 1$

$1 < \frac{1}{x^2} < \infty \Rightarrow 0 < \ln\frac{1}{x^2} < \infty \quad \therefore\ -\infty < \ln\left(\ln\left(\frac{1}{x^2}\right)\right) < \infty$

$\therefore \quad$ Range of $h(x)$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

(15-17) :

15. (b) 16. (a) 17. (a)

Since no point of the parabola is below x-axis

$\therefore \quad a^2 - 4 \le 0$

$\therefore \quad$ Maximum value of a is 2.

Equation of the parabola, when $a = 2$ is

$y = x^2 + 2x + 1$

It intersects y-axis at (0, 1)

Now, equation of the tangent at (0, 1) is $y = 2x + 1$

Since $y = 2x + 1$ touches the circle $x^2 + y^2 = r^2$

$\therefore \quad r = \frac{1}{\sqrt{5}}$

Equation of the tangent at (0, 1) to the parabola $y = x^2 + ax + 1$ is $ax - y + 1 = 0$ ...(i)

$\therefore \quad r = \frac{1}{\sqrt{a^2+1}}$

$\Rightarrow$ Radius is maximum when $a = 0$

$\therefore \quad$ Equation of the tangent is $y = 1$ (put $a = 0$ in (i))

$\therefore \quad$ Slope of the tangent is 0

Equation of the tangent is $y = ax + 1$, intercepts are $-1/a$ and 1

$\therefore \quad$ Area of the Δ bounded by tangent and the axes

$= \frac{1}{2}\left|-\frac{1}{a}.1\right| = \frac{1}{2|a|}$

It is minimum when $a = 2$

$\therefore$ Minimum area $= \frac{1}{4}$

18. (A) → (q); (B) → (s); (C) → (p); (D) → (r)

(A) Clearly, $f(x)$ is decreasing just before $x = 3$ and increasing after $x = 3$. For $x = 3$ to be the point of local maxima $f(3) \ge f(3 - 0)$

$\Rightarrow \quad -15 \ge 12 - 27 + \log_e(a^2 - 3a + 3)$

$\Rightarrow \quad 0 < a^2 - 3a + 3 \le 1 \Rightarrow a \in [1, 2]$.

(B) Let $f(x) = x + \cos x - a$

$\Rightarrow f'(x) = 1 - \sin x \ge 0 \ \forall\ x \in R$

Thus, $f(x)$ is increasing in $(-\infty, \infty)$.
For positive roots, $f(0) = 1 - a < 0 \Rightarrow a \in (1, \infty)$
(C) $f'(x) = -\sin x + a^2 \geq 0 \Rightarrow a^2 \geq \sin x \ \forall \ x \in R$
$\Rightarrow \quad a^2 \geq 1$
(D) Let $y = x^3 - 9x^2 + 24x + a = x(x^2 - 9x + 24) + a$

$$\frac{dy}{dx} = 3(x^2 - 6x + 8) = 3(x-4)(x-2)$$

For real roots of $f(x) = x^3 - 9x^2 + 24x + a$
$\therefore \quad f(2) \cdot f(4) < 0$

19. (A) → (p); (B) → (q); (C) → (r); (D) → (s)

(A) $(a + b)(c + d) \geq 81$ ...(i)

Now, $\frac{(a+b)+(c+d)}{2} \geq ((a+b)(c+d))^{1/2}$ [$\because$ A.M. $\geq$ G.M.]

$\therefore$ Minimum value of $a + b + c + d$ is 18.

(B) $\frac{(x+y)^{14}(1+xy)^{14}}{(xy)^{14}}$

Both factors in the numerator have 15 independent terms. So, total number of terms = 15×15 = 225.

(C) $(23)^{86} = (529)^{43} = (530 - 1)^{43}$
$= [(530)^{43} - {}^{43}C_1(530)^{42} + {}^{43}C_2(530)^{41} - \ldots. - {}^{43}C_{41}(530)^2] + (43 \times 530) - 1$

Hence the last two digit of $(23)^{86}$ are last two digits of $(43 \times 530) - 1$ *i.e.* 89.

(D) $T_3 = {}^5C_2 \frac{1}{x^3}(x^{\log_{10} x})^2 = 1000$

Taking log to the base 10 on both sides, we get
$-3\log_{10}x + 2(\log_{10}x)^2 - 2 = 0$
Put $\log_{10}x = t$

$\therefore \quad 2t^2 - 3t - 2 = 0 \Rightarrow \quad t = -\frac{1}{2}, 2$

$\because \quad x > 1$, so $\log_{10}x = 2 \Rightarrow x = 100$.

20. (3) : Let $x = 3\cos\theta$, $y = 3\sin\theta$,
$z = 2\cos\phi$ and $t = 2\sin\phi$
Now, $xt - yz = 6 \Rightarrow 6\cos\theta \sin\phi - 6\sin\theta \cos\phi = 6$
$\Rightarrow \sin(\phi - \theta) = 1 \Rightarrow \phi - \theta = 90° \Rightarrow \phi = 90° + \theta$
$\therefore \quad x = 3\cos\theta, \ y = 3\sin\theta$ and $z = -2\sin\theta, \ t = 2\cos\theta$
$P = xz = -6\sin\theta \cos\theta = -3\sin 2\theta$
Hence, $P_{max} = 3$.

21. (7) : $\left(\frac{n+1}{4}\right)(2^{n+1} - n - 2) = \sum_{k=1}^{n} k \cdot 2^{n+1-k} = S$ where

$S = 1.2^n + 2.2^{n-1} + 3.2^{n-2} + \ldots + n.2$ ...(i)

$\frac{1}{2}S = 2^{n-1} + 2.2^{n-2} + \ldots. + (n-1).2 + n$ ...(ii)

Subtracting (ii) from (i), we get

$\frac{1}{2}S = (2^n + 2^{n-1} + \ldots.. + 2) - n = 2(2^n - 1) - n$

$\therefore \quad S = 2(2^{n+1} - 2) - 2n = 2(2^{n+1} - n - 2)$

Hence, $\frac{n+1}{4} = 2 \Rightarrow n = 7$

22. (9) : Taking O as the origin, let the position vectors of A, B and C be $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively. Then the position vectors of G_1, G_2 and G_3 are

$\frac{\vec{b}+\vec{c}}{3}, \frac{\vec{c}+\vec{a}}{3}$ and $\frac{\vec{a}+\vec{b}}{3}$.

Now, $V_1 = \frac{1}{6}\left[\vec{a}\,\vec{b}\,\vec{c}\right]$ and $V_2 = \left[\overrightarrow{OG_1}, \overrightarrow{OG_2}, \overrightarrow{OG_3}\right]$

$\Rightarrow \quad V_2 = \frac{1}{27}[\vec{b}+\vec{c} \ \ \vec{c}+\vec{a} \ \ \vec{a}+\vec{b}] = \frac{2}{27}[\vec{a}\,\vec{b}\,\vec{c}]$

$\Rightarrow \quad V_2 = \frac{2}{27} \times 6V_1 \Rightarrow 9V_2 = 4V_1 \Rightarrow 4V_1/V_2 = 9$

UAV for disaster management by IIT Madras wins Microsoft challenge

Unmanned Aerial Vehicle (UAV) for Disaster Management by IIT Madras emerged winner of a challenge under Microsoft's Academia Accelerator programme on 5th February. The award-winning project aims to solve the issue of lack of systems for accessing accurate information by creating an end-to-end autonomous system, to provide precise information about where exactly the people are stuck, with the use of UAVs which are powered with AI and Computer Vision. Microsoft India hosted AXLE, a Microsoft Academia Accelerator an annual showcase of collaboration between Microsoft and academia.

The event brought together Computer Science faculty, Microsoft leaders and employees, several industry influencers and students from top

engineering colleges of India who showcased innovative ways of building state of the art technology to predict or manage natural disasters better.

For making a Mixed Reality app that makes basic tasks like communication, navigation and current status monitoring easy for rescuers, the team from IIT Guwahati was judged the first runner up in the competition, Microsoft said. The IIT Jodhpur team's Internet of Things (IoT)-based solution that acts as an early warning system and takes precautionary measures on detection of disasters was placed third in the competition. The three winning teams will be awarded ₹ 5 lakhs, ₹ 3 lakhs and ₹ 1 lakh respectively, Microsoft said.

The winning teams will receive technical and educational support through the AI for Earth grantee community and each winning team will also be awarded $5,000 in Azure credits from AI for Earth. Academia Accelerator Showcase builds on the best through mentor support, publishing support and pitch support provided and eventually provides the opportunity for the best among India's student developers to hone their CS skills further.

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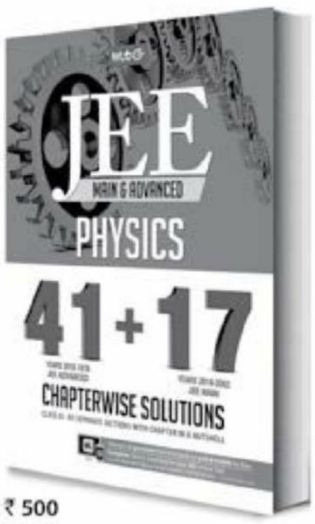

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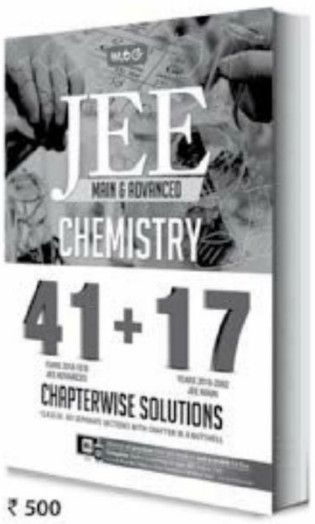

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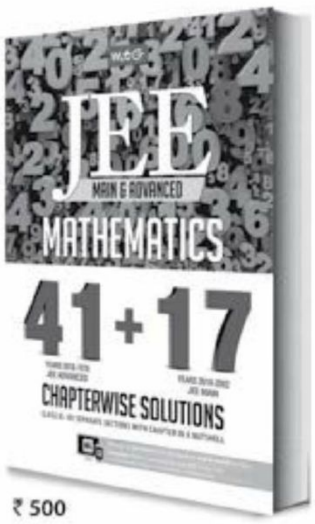

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MOCK TEST PAPER 2019
JEE MAIN

Exam Date : 7th to 20th April 2019

Time : 1 hr 15 min. **Series - 8**

The entire syllabus of Mathematics of JEE MAIN is being divided in to eight units, on each unit there will be a Mock Test Paper (MTP) which will be published in the subsequent issues. The syllabus for module break-up is given below:

Unit No.	Topic	Syllabus In Details
UNIT NO.8	Integral calculus	Integral as limit of a sum. Fundamental theorem of calculus. Properties of definite integrals. Evaluation of definite integrals, determining areas of the regions bounded by simple curves in standard form..
	Differential equation	Ordinary differential equations, their order and degree. Formation of differential equations. Solution of differential equations by the method of separation of variables, solution of homogeneous and linear differential equation
	Probability	Baye's theorem, Probability distribution of a random variate, Bernoulli trials and Binomial distribution

1. If $af(x)+bf\left(\frac{1}{x}\right)=\frac{1}{x}-5,\ x\neq 0,\ a\neq b$, then $\int_1^2 f(x)dx =$

(a) $\frac{(\ln 2-5)a+\frac{13}{2}b}{a^2-b^2}$ (b) $\frac{(\ln 2-5)a+\frac{7b}{2}}{a^2-b^2}$

(c) $\frac{(5-\ln 2)a+\frac{7b}{2}}{a^2-b^2}$ (d) none of these

2. The value of the integral $\int_0^{\pi/4}\frac{\sin x+\cos x}{3+\sin 2x}dx$ is

(a) log2 (b) log3

(c) $\frac{1}{4}\log 3$ (d) $\frac{1}{8}\log 3$

3. If $I_n=\int_0^{\pi/4}\tan^n x\,dx$, then $I_8+I_6=$

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{1}{6}$ (d) $\frac{1}{7}$

4. If for a real number y,$[y]$ is the greatest integer less than or equal to y, then the value of the integral $\int_{\pi/2}^{3\pi/2}[2\sin x]\,dx$, is

(a) $-\pi$ (b) 0 (c) $-\pi/2$ (d) $\pi/2$

5. $\int_0^{\pi/2}\frac{\sin^{2019}x}{\sin^{2019}x+\cos^{2019}x}dx$, is equal to

(a) $\pi/2$ (b) $\pi/4$ (c) $-\pi/2$ (d) $3\pi/4$

6. The value of the integral $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$, is

(a) $\pi^2/2$ (b) $\pi^2/4$ (c) $\pi^2/8$ (d) $\pi^2/16$

7. The least value of the function $\phi(x)=\int_{5\pi/4}^{x}(3\sin t+4\cos t)\,dt$ on the interval $[5\pi/4, 4\pi/3]$, is

(a) $\sqrt{3}+\frac{3}{2}$ (b) $-2\sqrt{3}+\frac{3}{2}+\frac{1}{\sqrt{2}}$

(c) $\frac{3}{2}+\frac{1}{\sqrt{2}}$ (d) none of these

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8. The value of the integral $\int_0^{\pi/2} \log|\tan x + \cot x|\,dx$

(a) $\pi\log 2$ (b) $-\pi\log 2$
(c) $\pi/2\log 2$ (d) none of these

9. The area of the region bounded by the parabola $(y-2)^2 = (x-1)$, the tangent to the parabola at the point (2, 3) and the x-axis is

(a) 3 (b) 6 (c) 9 (d) 12

10. The graphs of $f(x) = x^2$ and $g(x) = cx^3$ intersect at two points. If the area of the region bounded between $f(x)$ and $g(x)$ over the interval $\left(0, \frac{1}{c}\right)$ equal to $\frac{2}{3}$, then the value of $\frac{1}{c} + \frac{1}{c^2}$ is

(a) 20 (b) 2 (c) 6 (d) 12

11. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where c is a positive parameter, is of

(a) order 1, degree 3 (b) order 2, degree 3
(c) orders 1, degree 2 (d) none of these

12. The solution of the differential equation $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$, is

(a) $y = x\tan\left(\frac{x^2 + y^2 + C}{2}\right)$

(b) $x = y\tan\left(\frac{x^2 + y^2 + C}{2}\right)$

(c) $y = x\tan\left(\frac{C - x^2 - y^2}{2}\right)$

(d) none of these

13. Solution of the differential equation $\left(\frac{x+y-1}{x+y-2}\right)\frac{dy}{dx} = \left(\frac{x+y+1}{x+y+2}\right)$, given that $y = 1$ when $x = 1$, is

(a) $\log\left|\frac{(x-y)^2 - 2}{2}\right| = 2(x+y)$

(b) $\log\left|\frac{(x-y)^2 + 2}{2}\right| = 2(x-y)$

(c) $\log\left|\frac{(x+y)^2 + 2}{2}\right| = 2(x-y)$

(d) none of these

14. The equation of the curve passing through the origin and satisfying the differential equation $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$ is

(a) $(1 + x^2)y = x^3$ (b) $2(1 + x^2)y = 3x^3$
(c) $3(1 + x^2)y = 4x^3$ (d) none of these

15. The slope of the tangent at (x, y) to a curve passing through $\left(1, \frac{\pi}{4}\right)$ is given by $\frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$, then the equation of the curve is

(a) $y = \tan^{-1}\left\{\log\left(\frac{e}{x}\right)\right\}$ (b) $y = x\tan^{-1}\left\{\log\left(\frac{x}{e}\right)\right\}$

(c) $y = x\tan^{-1}\left\{\log\left(\frac{e}{x}\right)\right\}$ (d) none of these

16. If $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$, $y(1) = 1$, then one of the values of x_0 satisfying $y(x_0) = e$ is given by

(a) $\sqrt{2}e$ (b) $\sqrt{3}e$ (c) $\sqrt{5}e$ (d) $e/\sqrt{2}$

17. The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ satisfying $y(1) = 1$, is

(a) $y = xe^{x-1}$ (b) $y = x\ln x + x$
(c) $y = \ln x + x$ (d) $y = x\ln x + x^2$

18. The sum of squares of the perpendicular drawn from the points (0, 1) and (0, −1) to any tangent to a curve is 2. The equation of the curve is

(a) $2y = C(x + 2)$ (b) $y = C(x \pm 1)$
(c) $y = C(x+2)\frac{4}{3}$ (d) $y = C(x \pm 2)$

19. The function $y = f(x)$ is the solution of the differential equation $\frac{dy}{dx} + \frac{x}{x^2 - 1}y = \frac{x^4 + 2x}{\sqrt{1 - x^2}}$ in $(-1, 1)$ satisfying $f(0) = 0$. Then $\int_{-\sqrt{3}/2}^{\sqrt{3}/2} f(x)\,dx$ is equal to

(a) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ (b) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$
(c) $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$ (d) $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

20. Let $f: [1, \infty) \to [2, \infty)$ be a differentiable function such that $f(1) = 2$. If $6\int_1^x f(t)\,dt = 3xf(x) - x^3$ for all $x \geq 1$, then the value of $f(2)$ is

(a) 3 (b) 4 (c) 5 (d) 6

21. A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, 3 red balls and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls and 5 black balls. If two balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these two balls are drawn from box B_2 is

(a) $\frac{116}{181}$ (b) $\frac{126}{181}$ (c) $\frac{65}{181}$ (d) $\frac{55}{181}$

22. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is

(a) 7 (b) 9 (c) 8 (d) 5

23. If on an average one vessel in every 10 is wrecked, the probability that out of 5 vessels, at least 4 will arrive safely is

(a) 0.92 (b) 0.72
(c) 0.82 (d) none of these

24. There are 4 white and 3 black balls in a box. In another box there are 3 white and 4 black balls. An unbiased dice is rolled. If it shows a number less than or equal to 3 then a ball is drawn from the first box but if it shows a number more than 3 then a ball is drawn from the second box. If the ball drawn is black then the probability that the ball was drawn from the first box is

(a) 1/2 (b) 6/7 (c) 4/7 (d) 3/7

25. The probability distribution of a random variable is

$$P(X)=\begin{cases}\frac{X}{15}; X=1,2,3,4,5\\ 0, \text{ otherwise}\end{cases}.$$

Then $P\left\{\frac{1}{2}<X<\frac{5}{2}/X>1\right\}=$

(a) 1/5 (b) 1/7 (c) 2/15 (d) 1/15

26. In a box containing 100 bulbs, 10 are defective. What is the probability that out of a sample of 5 bulbs, none are defective?

(a) $\left(\frac{9}{10}\right)^5$ (b) $\frac{9}{10}$ (c) 10^{-5} (d) $\left(\frac{1}{2}\right)^2$

27. If X is a binomial variate with parameters n and p where $0<p<1$ such that $\frac{P(X=r)}{P(X=n-r)}$ is independent of n and r, then p equals

(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/5

28. If a random variable X follows binomial distribution with mean 2 and $E(x^2)=\frac{28}{5}$, then (n, p) =

(a) $\left(10,\frac{1}{5}\right)$ (b) $\left(5,\frac{1}{10}\right)$
(c) $\left(10,\frac{4}{5}\right)$ (d) none of these

29. The mean and variance of a binomial distribution are 4 and 3 respectively. Then probability of getting exactly six success in this distribution, is

(a) ${}^{16}C_6\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^{10}$ (b) ${}^{16}C_6\left(\frac{1}{4}\right)^{16}\left(\frac{3}{4}\right)^{20}$
(c) ${}^{16}C_6\left(\frac{1}{4}\right)^8\left(\frac{3}{4}\right)^{12}$ (d) ${}^{16}C_9\left(\frac{1}{4}\right)^{16}\left(\frac{3}{4}\right)^{20}$

30. For a fixed value of n, the maximum value of variance of binomial distribution is

(a) $n/4$ (b) $n/2$
(c) $n/8$ (d) none of these

SOLUTIONS

1. (b) : Here, $af(x)+bf\left(\frac{1}{x}\right)=\frac{1}{x}-5$ ...(i)

Replacing x by $\frac{1}{x}$ in (i), we get

$bf(x)+af\left(\frac{1}{x}\right)=x-5$ ...(ii)

Solving (i) and (ii), we get

$$f(x)=\frac{1}{a^2-b^2}\left\{\frac{a}{x}-bx-5a+5b\right\}$$

$$\therefore \int_1^2 f(x)\,dx=\frac{1}{a^2-b^2}\left[a\ln x-b\frac{x^2}{2}-5(a-b)x\right]_1^2$$

$$=\frac{1}{a^2-b^2}\left[(\ln 2-5)a+\frac{7b}{2}\right]$$

2. (c) : Let $I=\int_0^{\pi/4}\frac{\sin x+\cos x}{3+\sin 2x}dx$

$$=-\int_0^{\pi/4}\frac{\sin x+\cos x}{(\sin x-\cos x)^2-4}dx$$

Let $\sin x-\cos x=z \Rightarrow (\sin x+\cos x)dx=dz$

Now, when $x=0$, $z=-1$ and when $x=\pi/4$, $z=0$

$$\therefore I=-\int_{-1}^{0}\frac{dz}{z^2-2^2}=-\frac{1}{4}\left[\ln\left|\frac{z-2}{z+2}\right|\right]_{-1}^{0}+c$$

Therefore, $\int_0^{\pi/4}\frac{\sin x+\cos x}{3+\sin 2x}dx=\frac{1}{4}\ln 3$

3. (d) : Given that $I_n = \int_0^{\pi/4} \tan^n x\, dx$

$\therefore \quad I_8 + I_6 = \int_0^{\pi/4} (\tan^8 x + \tan^6 x)dx$

$= \int_0^{\pi/4} \tan^6 x(\tan^2 x + 1)\, dx = \int_0^{\pi/4} \tan^6 x\, d(\tan x)$

$= \left[\frac{\tan^7 x}{7}\right]_0^{\pi/4} = \frac{1}{7}$

4. (c) : Let $I = \int_{\pi/2}^{3\pi/2} [2\sin x]\, dx$

$= \int_{\pi/2}^{5\pi/6} [2\sin x]\, dx + \int_{5\pi/6}^{\pi} [2\sin x]\, dx + \int_{\pi}^{7\pi/6} [2\sin x]\, dx + \int_{7\pi/6}^{3\pi/2} [2\sin x]\, dx$

$= \int_{\pi/2}^{5\pi/6} 1\, dx + \int_{5\pi/6}^{\pi} 0\, dx + \int_{\pi}^{7\pi/6} (-1)\, dx + \int_{7\pi/6}^{3\pi/2} (-2)\, dx$

$= \left(\frac{5\pi}{6} - \frac{\pi}{2}\right) - \left(\frac{7\pi}{6} - \pi\right) - 2\left(\frac{3\pi}{2} - \frac{7\pi}{6}\right) = -\frac{\pi}{2}$

5. (b) : Let $I = \int_0^{\pi/2} \frac{\sin^{2019} x}{\sin^{2019} x + \cos^{2019} x}\, dx$ ...(i)

$= \int_0^{\pi/2} \frac{\sin^{2019}\left(\frac{\pi}{2} - x\right)}{\sin^{2019}\left(\frac{\pi}{2} - x\right) + \cos^{2019}\left(\frac{\pi}{2} - x\right)} dx$

$= \int_0^{\pi/2} \frac{\cos^{2019} x}{\cos^{2019} x + \sin^{2019} x}\, dx$ ...(ii)

Adding (i) and (ii), we get

$2I = \int_0^{\pi/2} \frac{\sin^{2019} x + \cos^{2019} x}{\sin^{2019} x + \cos^{2019} x}\, dx = [x]_0^{\pi/2} = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

6. (b) : Let $I = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\, dx = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}\, dx$

$= \pi\int_0^{\pi} \frac{\sin x\, dx}{1+\cos^2 x} - \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\, dx = \pi\int_0^{\pi} \frac{\sin x\, dx}{1+\cos^2 x} - I$

$\therefore \quad 2I = \pi\int_0^{\pi} \frac{\sin x\, dx}{1+\cos^2 x}$

$\Rightarrow \quad 2I = -\pi\int_1^{-1} \frac{dz}{1+z^2}$, where $z = \cos x$

$= -\pi[\tan^{-1} z]_1^{-1} = -\pi\left(-\frac{\pi}{4} - \frac{\pi}{4}\right) = \frac{\pi^2}{2}$

$\Rightarrow \quad I = \frac{\pi^2}{4}$

7. (b) : We have, $\phi(x) = \int_{5\pi/4}^{x} (3\sin t + 4\cos t)\, dt$

$\therefore \quad \phi'(x) = 3\sin x + 4\cos x$

In the interval $[5\pi/4, 4\pi/3]$, $\phi'(x) < 0$

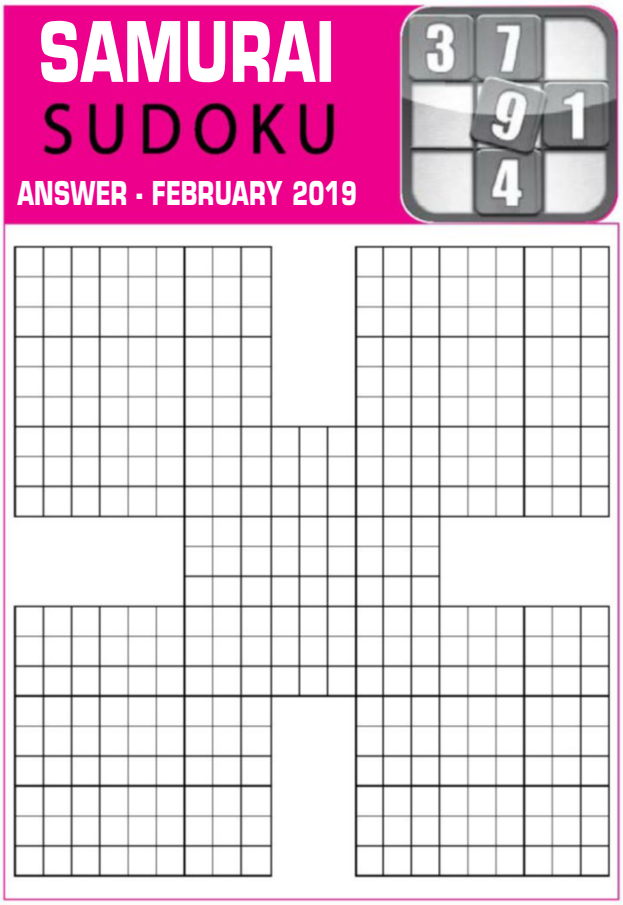
Therefore $\phi(x)$ is a decreasing function on $[5\pi/4, 4\pi/3]$.

$\Rightarrow$ $\phi(4\pi/3)$ is the least value of $\phi(x)$ on $[5\pi/4, 4\pi/3]$

Now, $\phi(4\pi/3) = \int_{5\pi/4}^{4\pi/3} (3\sin t + 4\cos t)dt$

$= [-3\cos t + 4\sin t]_{5\pi/4}^{4\pi/3} = \frac{3}{2} - 2\sqrt{3} + \frac{1}{\sqrt{2}}$

8. (a) : Let $I = \int_0^{\pi/2} \log|\tan x + \cot x|\, dx$



$$= \int_0^{\pi/2} \log \left| \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \right| dx$$

$$= -\int_0^{\pi/2} \log \sin x \, dx - \int_0^{\pi/2} \log \cos x \, dx = \pi \log 2$$

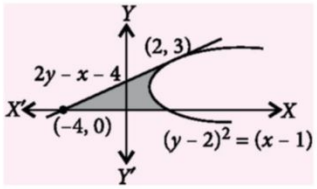
9. (c) : The equation of the parabola is

$y^2 - 4y - x + 5 = 0$

The equation of the tangent at (2, 3) is

$3y - 2(y+3) - \frac{x+2}{2} + 5 = 0$

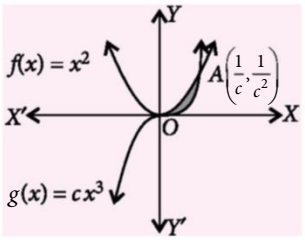
or $2y - x - 4 = 0$



$\therefore$ Required area $= \int_0^3 |\{(y-2)^2 + 1\} - \{2y - 4\}| \, dy$

$$= \int_0^3 (y^2 - 6y + 9) dy = \int_0^3 (3-y)^2 dy = -\left[\frac{(3-y)^3}{3} \right]_0^3 = 9$$

10. (c) : The two curves $f(x) = x^2$ and $g(x) = cx^3$ intersect at $A\left(\frac{1}{c}, \frac{1}{c^2}\right)$ and O (0, 0).



Given, $\int_0^{1/c} (x^2 - cx^3) dx = \frac{2}{3}$

$$\Rightarrow \left[\frac{x^3}{3} - \frac{cx^4}{4} \right]_0^{\frac{1}{c}} = \frac{2}{3} \Rightarrow \frac{1}{12c^3} = \frac{2}{3} \Rightarrow c = \frac{1}{2}$$

Hence $\frac{1}{c} + \frac{1}{c^2} = 2 + 4 = 6$

11. (a) : We have, $y^2 = 2c(x + \sqrt{c})$ ...(i)

$$\Rightarrow 2y \frac{dy}{dx} = 2c \Rightarrow y \frac{dy}{dx} = c \quad \text{...(ii)}$$

Eliminating c from (i) and (ii), we get

$$y^2 = 2y \frac{dy}{dx} \left(x + \sqrt{y \frac{dy}{dx}} \right) \Rightarrow y - 2x \frac{dy}{dx} = 2\sqrt{y} \left(\frac{dy}{dx} \right)^{3/2}$$

$$\Rightarrow \left(y - 2x \frac{dy}{dx} \right)^2 = 4y \left(\frac{dy}{dx} \right)^3$$

This is a differential equation of order 1 and degree 3.

12. (c) : We have, $x dx + y dy + \frac{x dy - y dx}{x^2 + y^2} = 0$

$$\Rightarrow \frac{1}{2} d(x^2 + y^2) + d\left(\tan^{-1} \frac{y}{x} \right) = 0$$

Integrating both sides, we get

$$\frac{1}{2}(x^2 + y^2) + \tan^{-1} \frac{y}{x} = \frac{C}{2} \Rightarrow \frac{C - x^2 - y^2}{2} = \tan^{-1} \frac{y}{x}$$

$$\Rightarrow y = x \tan \left(\frac{C - x^2 - y^2}{2} \right)$$

13. (d) : We have,

$$\left(\frac{x+y-1}{x+y-2} \right) \frac{dy}{dx} = \left(\frac{x+y+1}{x+y+2} \right) \quad \text{...(i)}$$

Put $x + y = z \Rightarrow 1 + \frac{dy}{dx} = \frac{dz}{dx}$ ...(ii)

Substituting (ii) in (i), we have

$$\left(\frac{z-1}{z-2} \right) \left(\frac{dz}{dx} - 1 \right) = \left(\frac{z+1}{z+2} \right)$$

$$\Rightarrow \left(\frac{dz}{dx} - 1 \right) = \left(\frac{z+1}{z+2} \right) \left(\frac{z-2}{z-1} \right)$$

$$\Rightarrow \frac{dz}{dx} = \frac{z^2 - z - 2}{z^2 + z - 2} + 1 \Rightarrow \frac{dz}{dx} = \frac{2z^2 - 4}{z^2 + z - 2}$$

$$\Rightarrow \frac{z^2 + z - 2}{z^2 - 2} dz = 2dx \Rightarrow \int \left(1 + \frac{z}{z^2 - 2} \right) dz = 2 \int dx$$

$$\Rightarrow z + \frac{1}{2} \log |z^2 - 2| = 2x + C$$

$$\Rightarrow (x + y) + \frac{1}{2} \log |(x+y)^2 - 2| = 2x + C$$

$$\Rightarrow 2(y - x) + \log |(x+y)^2 - 2| = 2C \quad \text{...(iii)}$$

Putting $x = 1$ and $y = 1$, we get $\log 2 = 2C$

From (iii) we get, $2(y - x) + \log|(x + y)^2 - 2| = \log 2$

14. (c) : We have, $(1 + x^2) \frac{dy}{dx} + 2xy = 4x^2$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{4x^2}{1+x^2}$$

Here, I.F. $= e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = 1 + x^2$

Thus, the general solution is

$$y(1 + x^2) = \int (1 + x^2) \frac{4x^2}{1+x^2} dx = 4 \int x^2 \, dx = \frac{4}{3} x^3 + C \quad \text{...(i)}$$

Since (i) is passing through (0, 0), therefore $C = 0$

Thus (i) becomes,

$$y(1 + x^2) = \frac{4}{3} x^3 \Rightarrow 3(1 + x^2) y = 4x^3$$

15. (c) : According to the problem,

$$\frac{dy}{dx} = \frac{y}{x} - \cos^2 \left(\frac{y}{x} \right)$$

Putting $\frac{y}{x}=v$, we get $\frac{dy}{dx}=v+x\frac{dv}{dx}$

$\Rightarrow \quad v+x\frac{dv}{dx}=v-\cos^2 v \quad \Rightarrow \quad \int \sec^2 v dv=-\int\frac{dx}{x}$

$\Rightarrow \quad \tan v=-\log|x|+C \Rightarrow \quad \tan\frac{y}{x}=-\log|x|+C \quad ...(i)$

Since (i) is passing through the point $(1, \pi/4)$

$\therefore \quad \tan\frac{\pi}{4}=-\log 1+C \Rightarrow C=1$

Now (i) becomes, $\tan\frac{y}{x}=-\log|x|+1=\log\left(\frac{e}{x}\right)$

$\Rightarrow \quad y=x\tan^{-1}\left\{\log\left(\frac{e}{x}\right)\right\}$, which is the required equation of the curve.

16. (b) : The given differential equation is

$$\frac{dy}{dx}=\frac{xy}{x^2+y^2},\ y(1)=1$$

Putting $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\Rightarrow \quad v+x\frac{dv}{dx}=\frac{v}{1+v^2} \quad \Rightarrow \quad x\frac{dv}{dx}=\frac{v}{1+v^2}-v=\frac{-v^3}{1+v^2}$

$\Rightarrow \quad \int\frac{1+v^2}{v^3}dv=-\int\frac{dx}{x} \Rightarrow -\frac{1}{2v^2}+\log|v|=-\log|x|+C$

$\Rightarrow \quad -\frac{x^2}{2y^2}+\log\left|\frac{y}{x}\right|+\log|x|=C \quad ...(i)$

Putting $x = 1$, $y = 1$ in (i), we get $C = -1/2$

Now, (i) becomes $\log y-\frac{x^2}{2y^2}=-\frac{1}{2} \quad ...(ii)$

Putting $x = x_0$, $y = e$ in (ii), we get $x_0=\sqrt{3}e$

17. (b) : The given equation is

$$\frac{dy}{dx}=\frac{x+y}{x} \quad \Rightarrow \quad \frac{dy}{dx}-\frac{y}{x}=1$$

Here I.F. $= e^{-\int\frac{dx}{x}}=e^{-\log x}=e^{\log x^{-1}}=x^{-1}=1/x$

Therefore the general solution is

$$\frac{y}{x}=\int\frac{dx}{x}=\log|x|+C \quad ...(i)$$

Putting $x = 1$ and $y = 1$, we get $C = 1$

Thus, (i) becomes $y = x \log|x| + x$

18. (b) : The equation of any tangent to a curve $y = f(x)$ is

$$y-y_1=\frac{dy}{dx}(x-x_1) \quad \Rightarrow \quad x\frac{dy}{dx}-y+\left(y_1-x_1\frac{dy}{dx}\right)=0$$

According to the problem, we have

$$\left|\frac{-1+y_1-x_1\frac{dy}{dx}}{\sqrt{\left(\frac{dy}{dx}\right)^2+1}}\right|^2+\left|\frac{1+y_1-x_1\frac{dy}{dx}}{\sqrt{\left(\frac{dy}{dx}\right)^2+1}}\right|^2=2$$

$$\Rightarrow \quad \left\{\left(y_1-x_1\frac{dy}{dx}\right)-1\right\}^2+\left\{\left(y_1-x_1\frac{dy}{dx}\right)+1\right\}^2=2\left\{\left(\frac{dy}{dx}\right)^2+1\right\}$$

$$\Rightarrow \quad 2\left\{\left(y_1-x_1\frac{dy}{dx}\right)^2+1\right\}=2\left\{\left(\frac{dy}{dx}\right)^2+1\right\}$$

$$[\because\ (a-b)^2+(a+b)^2=2(a^2+b^2)]$$

$$\Rightarrow \quad \left(y_1-x_1\frac{dy}{dx}\right)^2=\left(\frac{dy}{dx}\right)^2$$

$\Rightarrow \quad y-x\frac{dy}{dx}=\pm\frac{dy}{dx}$ [writing x_1 as x and y_1 as y]

$\Rightarrow \quad \frac{1}{x\pm1}dx=\frac{1}{y}dy \quad \Rightarrow \quad \log(x\pm1)=\log y+\log K$

$\Rightarrow \quad Ky = x \pm 1 \quad \Rightarrow \quad y = C(x \pm 1)$

19. (b) : We have, $\frac{dy}{dx}+\frac{xy}{x^2-1}=\frac{x^4+2x}{\sqrt{1-x^2}}$

I.F. $=e^{\int\frac{xdx}{x^2-1}}=e^{\frac{1}{2}\log|x^2-1|}=e^{\frac{1}{2}\log(1-x^2)}=\sqrt{1-x^2}$

Therefore the general equation is

$$y\sqrt{1-x^2}=\int\frac{x^4+2x}{\sqrt{1-x^2}}\times\sqrt{1-x^2}\,dx$$

$\Rightarrow \quad y\sqrt{1-x^2}=\int(x^4+2x)\,dx$

PUZZLE CORNER

ANSWER · FEBRUARY 2019

6+ 3	1	4 4	18+ 6	5	2
2	17+ 4	6	12× 3	3× 1	5
1− 6	5	2	4	3	1
5	1− 2	3	24× 1	6	4
12× 4	3	11+ 1	5	3+ 2	6
5− 1	6	5	9+ 2	4	3

$\Rightarrow \quad y\sqrt{1-x^2} = \frac{x^5}{5} + x^2 + C$ ...(i)

Putting $x = 0$ and $y = 0$ in (i), we get $C = 0$

$\therefore \quad y\sqrt{1-x^2} = \frac{x^5}{5} + x^2 \Rightarrow f(x) = \frac{x^5}{5\sqrt{1-x^2}} + \frac{x^2}{\sqrt{1-x^2}}$

So, $\int_{-\sqrt{3}/2}^{\sqrt{3}/2} f(x)dx = \int_{-\sqrt{3}/2}^{\sqrt{3}/2} \frac{x^5}{5\sqrt{1-x^2}}dx + \int_{-\sqrt{3}/2}^{\sqrt{3}/2} \frac{x^2}{\sqrt{1-x^2}}dx$

$= 2\int_0^{\sqrt{3}/2} \frac{x^2}{\sqrt{1-x^2}}dx \qquad \left[\because \int_{-\sqrt{3}/2}^{\sqrt{3}/2} \frac{x^5 dx}{5\sqrt{1-x^2}} = 0\right]$

$= 2\int_0^{\pi/3} \sin^2\theta\, d\theta$, where $x = \sin\theta$

$\Rightarrow \int_{-\sqrt{3}/2}^{\sqrt{3}/2} f(x)\,dx = \int_0^{\pi/3} (1-\cos 2\theta)\,d\theta$

$= \left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\pi/3} = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$

20. (d) : We have, $6\int_1^x f(t)\,dt = 3xf(x) - x^3$

Differentiating both sides with respect to x, we get

$6f(x) = 3f(x) + 3xf'(x) - 3x^2$

$\Rightarrow \quad 3f(x) = 3xf'(x) - 3x^2$

$\Rightarrow \quad f'(x) - \frac{1}{x}f(x) = x \quad \Rightarrow \quad \frac{d}{dx}\{f(x)\} + \left(-\frac{1}{x}\right)f(x) = x$

Here I.F. $= e^{\int -\frac{1}{x}dx} = e^{-\log x} = 1/x$

$\therefore \quad$ The general solution is $\frac{f(x)}{x} = \int x \times \frac{1}{x}dx = x + C$

$\Rightarrow \quad f(x) = x^2 + Cx$

$\therefore \quad f(1) = 1 + C \quad \Rightarrow \quad 2 = 1 + C \quad \Rightarrow \quad C = 1$

Thus, $f(x) = x^2 + x \quad \therefore \quad f(2) = 4 + 2 = 6$

21. (d) : Let us define the events in the following way :

E_1: selection of box B_1, E_2: selection of box B_2,

E_3: selection of box B_3

A: selection of one white and one red ball

$\therefore \quad P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

and $P(A/E_1) = \frac{{}^1C_1 \times {}^3C_1 \times {}^2C_0}{{}^6C_2} = \frac{1}{5}$

$P(A/E_2) = \frac{{}^2C_1 \times {}^3C_1 \times {}^4C_0}{{}^9C_2} = \frac{1}{6}$

$P(A/E_3) = \frac{{}^3C_1 \times {}^4C_1 \times {}^5C_0}{{}^{12}C_2} = \frac{2}{11}$

= Required probability $= P(E_2/A)$

$= \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$

$= \frac{\frac{1}{3} \times \frac{1}{6}}{\frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{2}{11}} = \frac{55}{181}$

22. (a) : Let the coin be tossed n times and X be the random variable which denotes number of heads obtained in n tosses of the coin. Then

$P(X = r) = {}^nC_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r} = {}^nC_r \left(\frac{1}{2}\right)^n ; r = 0, 1, 2..., n$

Given that $P(X \geq 2) \geq 0.96 \Rightarrow \{1 - P(X < 2)\} \geq 0.96$

$\Rightarrow \quad \{1 - P(X = 0) - P(X = 1)\} \geq 0.96$

$\Rightarrow \quad \left(1 - {}^nC_0\left(\frac{1}{2}\right)^n - {}^nC_1\left(\frac{1}{2}\right)^n\right) \geq 0.96$

$\Rightarrow \quad \frac{1}{2^n}(2^n - {}^nC_0 - {}^nC_1) \geq 0.96 \Rightarrow 1 - \frac{n+1}{2^n} \geq 0.96$

$\Rightarrow \quad (n + 1) \leq (0.04)2^n \quad \Rightarrow \quad n = 8, 9, 10, ...$

Hence the least value of n is 8.

23. (a) : Given, probability of vessel being wrecked = 1/10

Then, probability of a vessel arriving safe = 9/10

$\therefore \quad$ Required probability

$= {}^5C_4\left(\frac{9}{10}\right)^4 \times \frac{1}{10} + \left(\frac{9}{10}\right)^5 = \frac{9^4 \times 14}{10^5} = 0.92$

24. (d) : Let us define the events in the following way:

E_1: The ball is drawn from the first box

E_2: The ball is drawn from the second box

A: The drawn ball is black.

$\therefore \quad P(E_1) = \frac{3}{6} = \frac{1}{2}$ and $P(E_2) = \frac{3}{6} = \frac{1}{2}$

Clearly, E_1 and E_2 are mutually exclusive and exhaustive.

Now, $P(A/E_1) = \frac{3}{7}$ and $P(A/E_2) = \frac{4}{7}$.

$\therefore \quad P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$

$= \frac{\frac{1}{2} \cdot \frac{3}{7}}{\frac{1}{2} \cdot \frac{3}{7} + \frac{1}{2} \cdot \frac{4}{7}} = \frac{3}{7}$

25. (b) : We have, $P(X)=\begin{cases}\frac{X}{15}; X=1,2,3,4,5\\ 0, \text{ otherwise}\end{cases}$

$\therefore \quad P(X=1 \text{ or } 2)=P(X=1)+P(X=2)=\frac{1}{15}+\frac{2}{15}=\frac{1}{5}$

Now $P\left(\frac{1}{2}<X<\frac{5}{2}/X>1\right)=\frac{P\left\{\left(\frac{1}{2}<X<\frac{5}{2}\right)\cap X>1\right\}}{P(X>1)}$

$=\frac{P\{(X=1 \text{ or } 2)\cap(X>1)\}}{P(X>1)}=\frac{P(X=2)}{1-P(X=1)}=\frac{\frac{2}{15}}{1-\frac{1}{15}}=\frac{1}{7}.$

26. (a) : We have, $P(\text{defected bulb})=\frac{10}{100}=\frac{1}{10}$

$\therefore \quad P(\text{non-defective bulb}) = 9/10$

Let X be the random variable, that denotes number of defective bulb. Here $n = 5$

$\therefore \quad P(X=r)={}^5C_r\left(\frac{1}{10}\right)^r\left(\frac{9}{10}\right)^{5-r}, \ r=0,1,...,5$

So, $P(X=0)={}^5C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^5=\left(\frac{9}{10}\right)^5$

27. (a) : We have, $\frac{P(X=r)}{P(X=n-r)}=\frac{{}^nC_r p^r(1-p)^{n-r}}{{}^nC_{n-r}p^{n-r}(1-p)^r}$

$\Rightarrow \quad \frac{P(X=r)}{P(X=n-r)}=\left(\frac{1-p}{p}\right)^{n-2r}=\left(\frac{1}{p}-1\right)^{n-2r}$

For this to be independent of n and r, we must have

$\frac{1}{p}-1=1 \ \Rightarrow \ p=\frac{1}{2}.$

28. (a) : Let a random variable X follows a binomial distribution with parameters n and p .

Given, mean $=E(x)=np=2$ and $E(x^2)=\frac{28}{5}.$

$\therefore \quad npq=E(x^2)-[E(x)]^2=\frac{28}{5}-2^2=\frac{8}{5}.$

Now, $\frac{npq}{np}=\frac{8}{5}\div 2=\frac{4}{5} \ \Rightarrow \ q=\frac{4}{5}$

$\therefore \quad p=\frac{1}{5}$ and hence $n=10$

29. (a) : Given that $X \sim B(n, p)$ with $np = 4$ and $npq = 3$

$\Rightarrow \quad q=\frac{3}{4}, p=\frac{1}{4}$ and $n=16$

$\therefore \quad P(X=r)={}^{16}C_r\left(\frac{1}{4}\right)^r\left(\frac{3}{4}\right)^{16-r}, \ r=0,1,...,16$

$\Rightarrow \quad P(X=6)={}^{16}C_6\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^{10}$

30. (a) : Variance of Binomial distribution = npq

$=n\left[\left(\frac{p+q}{2}\right)^2-\left(\frac{p-q}{2}\right)^2\right]$

$=n\left[\frac{1}{4}-\left(\frac{p-q}{2}\right)^2\right]\le\frac{n}{4} \quad \left[\because\left(\frac{p-q}{2}\right)^2\ge 0\right]$

Hence, for a fixed value of n, the variance of the binomial distribution can never exceed $n/4$. So, the maximum value of variance is $n/4$.

JEE Main April 2019 Notification

The National Testing Agency (NTA) has conducted the first JEE Main Examination during January 8 to 12, 2019 and the result of the same has been declared on January 19th, 2019 (for Paper-1) and January 31st, 2019 (for Paper-2). Now, the NTA announces to conduct JEE Main April 2019 Examination for admission to Undergraduate Programs in NITs, IIITs and other Centrally Funded Technical Institutions (CFTIs), etc. between **7th April 2019 to 20th April 2019.**

Those candidates who have already appeared in JEE Main January 2019 Examination can appear in JEE Main April 2019 Examination for improvement, if they so wish. The candidates who could not appear in the JEE Main January 2019 Examination, may also appear in JEE Main April 2019 Examination. The Student's best of the two NTA scores will be considered for preparation of Merit List/Ranking.

The test details are given below:

Paper	Subjects	Mode of Examination	Timing of Examination	
			First Shift	Second Shift
Paper-1 (For B.E./ B.Tech.)	Mathematics, Physics & Chemistry	"Computer Based Test (CBT)" mode only	09.30 a.m. to 12.30 p.m.	02.30 p.m. to 05.30 p.m.
Paper-2 (For B.Arch/ B. Planning)	Mathematics – Part I Aptitude Test – Part II	"Computer Based Test (CBT)" mode only	Can be held in two or more shifts	
	Drawing Test – Part III	"Pen & Paper Based" (offline) mode to be attempted on Drawing Sheet		

A candidate may appear in Paper-1 and/or Paper-2 depending upon the course/s he/she wishes to pursue. All the candidates aspiring to take admission to the Undergraduate Programs at IITs for the year 2019 will have to appear in the Paper-1 (B. E. /B. Tech.) of JEE Main 2019. Based on the performance in Paper-1 (B.E./B.Tech.) of JEE Main 2019, number of top candidates as per the requirement of JEE Advanced (including all categories) will be eligible to appear in JEE Advanced 2019. Candidates who desire to appear in the test may see the detailed Information Bulletin for JEE Main April 2019 available on the website ***www.nta.ac.in*** and ***www.jeemain.nic.in***.

In order to appear in JEE Main April 2019 Examination the candidates are required to apply only online between **8th February 2019** to **07th March 2019**. The fees can be paid online up to **8th March 2019**.

MATH archives

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of JEE (Main & Advanced) Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for JEE (Main & Advanced). In every issue of MT, challenging problems are offered with detailed solution. The reader's comments and suggestions regarding the problems and solutions offered are always welcome.

1. A twice differentiable function $f(x)$ is defined for all real numbers and satisfies the conditions as $f(0) = 2$, $f'(0) = -5$ and $f''(0) = 3$. The function $g(x)$ is defined by $g(x) = e^{ax} + f(x)\ \forall\ x \in R$, where a is any constant. If $g'(0) + g''(0) = 0$, then sum of all possible values of a is

(a) 1 (b) −1 (c) 2 (d) 0

2. Points P, Q, R lie on same line. Three semi circles with the diameters PQ, QR, PR are drawn on same side of line segment PR. The centres of the semicircles are A, B, O respectively. A circle with centre C touches all 3 semi circles then the radius of this circle is $(AQ = a, BQ = b)$

(a) $\frac{ab}{a+b}$ (b) $\frac{ab(a+b)}{a^2+b^2}$

(c) $\frac{ab(a+b)}{a^2+ab+b^2}$ (d) $\frac{ab(a+b)}{(a-b)^2}$

3. S_1 : Contrapositive of "If the weather is cold, then it will snow" is "If it will not snow, then the weather is not cold"

S_2 : Negation of "If it snows then they do not drive the car" is "It snows and they drive the car"

Which of the following is true, regarding the above statements?

(a) Only S_1 (b) Both S_1, S_2

(c) Neither S_1 nor S_2 (d) Only S_2

4. Find the smallest integer value of P for which the function $f(x) = 6Px - P\sin 4x - 5x - \sin 3x$ is monotonic increasing and has no critical point on R.

(a) 5 (b) 4 (c) 3 (d) 6

5. If for defined real values of α, $\cot^3\alpha + \cot^2\alpha + \cot\alpha = 1$ then $\cos 2\alpha - \tan 2\alpha =$ .....

(a) 5 (b) 1 (c) 0 (d) −1

6. If $\vec{x} = 3\hat{i} - 6\hat{j} - \hat{k}$, $\vec{y} = \hat{i} + 4\hat{j} - 3\hat{k}$ and $\vec{z} = 3\hat{i} - 4\hat{j} - 12\hat{k}$, then the magnitude of the scalar projection of $\vec{x} \times \vec{y}$ on $\vec{z}$ is

(a) 12 (b) 15 (c) 14 (d) 13

7. Let $\pi_1 : x - y\sin\alpha - z\sin\beta = 0$, $\pi_2 : x\sin\alpha - y + z\sin\gamma = 0$, $\pi_3 : x\sin\beta + y\sin\gamma - z = 0$, be 3 planes where $\sin\alpha$, $\sin\beta$, $\sin\gamma \neq 0$

S_1: $\pi_1 \perp \pi_2$ if $2\sin\alpha = \sin\beta\ \sin\gamma$

S_2 : If π_1, π_2, π_3 are mutually perpendicular then $\sin\alpha \cdot \sin\beta \cdot \sin\gamma = 3$.

S_3: If $\alpha + \beta + \gamma = \frac{\pi}{2}$, then π_1, π_2, π_3 have a common line of intersection.

S_4: If $\alpha + \beta + \gamma = \frac{\pi}{2}$, then π_1, π_2, π_3 can never meet in a common line.

Then find the number of statements which are correct.

(a) 1 (b) 2 (c) 4 (d) 3

8. Let $f(x)$ be a differentiable non-decreasing function such that $\int_0^x (f(t))^3\,dt = \frac{1}{x^2}\left(\int_0^x f(t)\,dt\right)^3\ \forall x \in R - \{0\}$ and $f(1) = 1$. If $\int_0^x f(t)\,dt = g(x)$ then value of $\frac{xg'(x)}{g(x)}$

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(a) can be equal to 1 (b) can be equal to 2
(c) can be equal to –1 (d) not independent of x

9. Let $f(x)$ be a real valued function defined on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that

$$f(x) = e^{2x} + \int_0^{\sin x} \frac{e^t}{\cos^2 x + 2t\sin x - t^2}\,dt, \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right).$$

Let $g(x)$ be the inverse of $f(x)$ where $0 \le x < \pi/2$ then the value of $f(0) + f'(0) + f''(0)$ equals
(a) $g'(1)$ (b) $3g'(1)$
(c) $(g'(1))^{-2}$ (d) $(g'(1))^{-1}$

10. If the system of linear equations $x + 2ay + az = 0$, $x + 3by + bz = 0$, $x + 4cy + cz = 0$ has a non-trivial solution and a, b and $c > 0$ then the minimum value of $\left(\frac{a^3 + c^3}{b^3}\right)$ is

(a) 1 (b) 0 (c) 2 (d) 3

SOLUTIONS

1. (b) : Here $g'(0) = a - 5$, $g''(0) = a^2 + 3$
$\because$ $g'(0) + g''(0) = 0$
$\Rightarrow$ $(a + 2)(a - 1) = 0 \Rightarrow a = 1, -2$

2. (c) : $AC = a + c$, $BC = b + c$, $OA = b$, $OB = a$, $OC = a + b - c$. Use cosine rule and ΔABC and ΔOAC of ΔABC and ΔOAC

3. (b) **4. (a)**

5. (b) : $\cot\alpha(\cot^2\alpha + 1) = 1 - \cot^2\alpha$

$$\Rightarrow \cot\alpha \operatorname{cosec}^2\alpha = \frac{\sin^2\alpha - \cos^2\alpha}{\sin^2\alpha}$$

$\Rightarrow$ $\cot\alpha = -\cos 2\alpha$

Now, $\cos 2\alpha - \tan 2\alpha$

$$= -\cot\alpha - \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{-1}{\tan\alpha} - \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$= \frac{-1 + \tan^2\alpha - 2\tan^2\alpha}{\tan\alpha(1-\tan^2\alpha)} = \frac{-(1+\tan^2\alpha)}{1-\tan^2\alpha}\cot\alpha$$

$$= \frac{1}{-\cos 2\alpha}\cdot\cot\alpha = \frac{1}{\cot\alpha}\cdot\cot\alpha = 1$$

6. (c) **7. (b)** **8. (a)**

9. (c) : We have, $f(g(x)) = x$

$\Rightarrow$ $f'(g(x))\,g'(x) = 1 \Rightarrow f'(g(1))\,g'(1) = 1$

$\Rightarrow$ $f'(0)g'(1) = 1 \Rightarrow g'(1) = \frac{1}{3}$ $[\because f'(0) = 3]$

$\therefore$ Required value $= 1 + 3 + 5 = 9 = \frac{1}{(g'(1))^2}$

10. (c) : $\Delta = 0 \Rightarrow b = \frac{2ac}{a+c}$

Now, $t = \frac{a^3 + c^3}{b^3} = \frac{(a^3 + c^3)}{8a^3c^3} \times (a+c)^3$

$$= \frac{(a+c)^4(a^2 - ac + c^2)}{8a^3c^3} \quad \text{....(i)}$$

Now, $(a - c)^2 = a^2 - 2ac + c^2 \ge 0$
$\Rightarrow$ $a^2 - ac + c^2 \ge ac$ ....(ii)

Also, $a + c \ge 2\sqrt{ac} \Rightarrow (a+c)^4 \ge 16a^2c^2$ ...(iii)

Multiplying (ii) and (iii), we get

$$(a + c)^4 (a^2 - ac + c^2) \ge 16a^3c^3$$

$$\Rightarrow \frac{(a+c)^4(a^2 - ac + c^2)}{8a^3c^3} \ge 2 \Rightarrow t \ge 2$$

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

6				1				3
		1		8		6		
	8						1	
			8		5			
3	2					5	7	
			3		2			
	4					8		
		9		2		1		
7				9			2	

3				8				9
		8		7		5		
	2						1	
			1		8			
4	1						5	2
			5		7			
							9	
		2		6		1		
8				5				6

				5				
			9				1	
				2			5	
							6	
4				8				1
		6		3		8		
	8					6		
			5		9			
1	7					5	9	
			4		3			
	1					2		
		4		9		1		
8				2				7

			2					8
		1		2			3	
	3					8		
				2			1	
9	1						4	2
			4		7			
				5			8	
		7		8		5		
4				7				1

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

Challenging PROBLEMS ON JEE Advanced

Exam on 19th May, 2019

Single Option Correct Type

1. Let $P(x)=x^2+\frac{x}{2}+b$ and $Q(x) = x^2 + cx + d$ be two polynomials with real coefficients such that $P(x)Q(x) = Q(P(x))$ for all real x. The number of real roots of $P(Q(x)) = 0$ are

(a) 0 (b) 2
(c) 4 (d) data insufficient

2. It is given that the curve $y = |\sin x|$ intersects the line $y = kx (k > 0)$ at exactly three points $x = \alpha, \beta, \gamma$ where $\alpha < \beta < \gamma$. Then $\frac{\cos\gamma}{\sin\gamma+\sin 3\gamma} =$

(a) $\frac{1+\gamma^2}{4\gamma}$ (b) $\frac{\gamma^2-1}{4\gamma}$ (c) $\frac{4\gamma}{1+\gamma^2}$ (d) $\frac{4\gamma}{\gamma^2-1}$

3. Define the sequence a_n recursively as follows: $a_0 = 0$, $a_1 = 1$ and for $n \geq 2$, a_n is the remainder of $a_{n-1} + a_{n-2}$ when divided by 3. The numerical value of $\sum_{k=2017}^{2024} a_k$ is

(a) 6 (b) 7 (c) 8 (d) 9

4. Given line $L : x + y = 9$, circle $M : 2x^2 + 2y^2 - 8x - 8y = 1$. Point A is on L, B, C are on M such that $\angle BAC = 45°$ and line AB passes through center of M. The range of x-co-ordinate of point A is

(a) [2, 5] (b) [3, 6] (c) [4, 7] (d) [5, 8]

5. Suppose a quadratic function $f(x) = ax^2 + bx + c$ $(a \neq 0)$, $(a, b, c \in R)$ satisfies the following three conditions

(i) $f(x - 4) = f(2 - x)$ and $f(x) \geq x$, $\forall\ x \in R$

(ii) When $x \in (0, 2)$, $f(x) \leq \left(\frac{x+1}{2}\right)^2$

(iii) The minimum value of $f(x)$ on R is 0.

The possible value of $4(f(3) - f(2))$ is

(a) 4 (b) 5 (c) 6 (d) 7

6. For $\theta \in (0, \pi)$, which of the following statement is correct?

(a) $\int_0^\theta \sqrt{1+\cos^2 t}\ dt > \sqrt{\theta^2+\sin^2\theta}$

(b) $\int_0^\theta \sqrt{1+\cos^2 t}\ dt < \sqrt{\theta^2+\sin^2\theta}$

(c) $\int_0^\theta \sqrt{1+\cos^2 t}\ dt = \sqrt{\theta^2+\sin^2\theta}$

(d) $\int_0^\theta \sqrt{1+\cos^2 t}\ dt = \frac{1}{2}\sqrt{\theta^2+\sin^2\theta}$

7. If $f(x)=\lim_{m\to\infty}\frac{\log(x^m+e^m)}{m}$, $x \geq 0$, $m \in N$, then

(a) $f(x)$ is discontinuous on $[0, \infty)$

(b) $f(x)$ is non-differentiable on $[0, \infty)$

(c) Area bounded by $f(x)$ between $x = 0$, $x = e$ and x-axis is 2 units

(d) $f(x)$ is periodic in $[0, \infty)$

8. A plane Π is given and on one side of the plane, lies three non-collinear points A, B, C such that the plane determined by them is not parallel to Π. Three random points A', B', C' in Π are selected. Let L, M, N be the midpoints of AA', BB' and CC' and let G be the centroid of ΔLMN. The locus of G as A', B', C' varies across Π is

(a) a plane parallel to Π

(b) a plane parpendicular to Π

(c) a line parallel to Π

(d) a line perpendicular to Π.

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9. Let a, b, c be real numbers and z_1, z_2, z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = 1$ and $\frac{z_1}{z_2}+\frac{z_2}{z_3}+\frac{z_3}{z_1}=1$, then a possible value of $|az_1 + bz_2 + cz_3|$ is

(a) $\sqrt{(a+b)^2+c^2}$ (b) $\sqrt{(a-b)^2+c^2}$

(c) $\sqrt{(a-b)^2-c^2}$ (d) $\sqrt{(a+b)^2-c^2}$

10. If $f(x)$ is monotonic, differentiable real valued function and $I_1=\int_a^b [f(x)+f(a)][f(x)-f(a)]dx$,

$I_2=\int_{f(a)}^{f(b)} x[b-f^{-1}(x)]dx$ $(a, b \in R)$, then $\frac{I_1}{I_2}=$

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

Comprehension Type

Paragraph for Q. No. 11 and 12

For $x \geq 0$, let $f(x)$ represent the only real root of the equation $y^3 + 26xy = 27$.

11. On [0, 1], the function $f(x)$ is

(a) strictly increasing (b) strictly decreasing

(c) non-decreasing (d) non-increasing

12. If $\int_0^1 f^2(x)\cdot \log f(x)dx=\frac{1}{k}(243\log 3-128)$, then $k =$

(a) 50 (b) 52 (c) −50 (d) −52

Paragraph for Q. No. 13 and 14

A chess match between two players A and B is won by whoever first wins a total of 2 games. A's chances of winning, drawing or losing any particular game are a, b, c respectively. The games are independent and $a + b + c = 1$.

13. The probability that A wins match after $(n + 1)$ games $(n \geq 1)$ is

(a) $a^2(nb^{n-1} + n(n-1)c \cdot b^{n-2})$

(b) $a^2(n(n-1)b^{n-1} + n(n-2)c^n)$

(c) $c^2(nb^{n-1} + n(n-1) \cdot a \cdot b^{n-2})$

(d) $c^2(n(n-1)b^{n-1} + n(n-2)a^n)$

14. The probability that B wins the match after the 4^{th} game is

(a) $3bc^2(b + 2a)$ (b) $3ac^2(b + 2a)$

(c) $3ac^2(b + 2a)$ (d) $3b^2c(b + 2a)$

Integer Answer Type

15. If $A=\begin{bmatrix}-1 & 1\\ 0 & -2\end{bmatrix}=B^3+C^3$, where B and C are 2×2 matrix, then value of $|Tr(B) + Tr(C)| =$

16. Consider a ΔABC formed by the points 't_1', 't_2', 't_3' on the parabola $y^2 = 4ax$. If the orthocenter of the Δ is the focus of the parabola, then $(t_1 - 1)(t_2 - 1)(t_3 - 1) =$

17. The possible values of $\sqrt{x^2+x+1}-\sqrt{x^2-x+1}$ as x ranges over all real numbers is an interval (a, b), then $|b - a| =$

18. The number of real solutions of the equation $\cos(\cos(\cos(\cos x))) = \sin(\sin(\sin(\sin x)))$ is/are

19. Let $f : (0, \infty) \to R$ be a twice continuously differentiable function such that $|f''(x) + 2xf'(x) + (x^2 + 1)f(x)| \leq 1$ for all x, then $\lim_{x\to\infty} f(x) =$

20. Given $I_n=\int_0^1 (1+x+x^2+...+x^{n-1})\,(1+3x+5x^2+....+(2x-1)x^{n-1})dx$, then $\sqrt{I_9} =$

SOLUTIONS

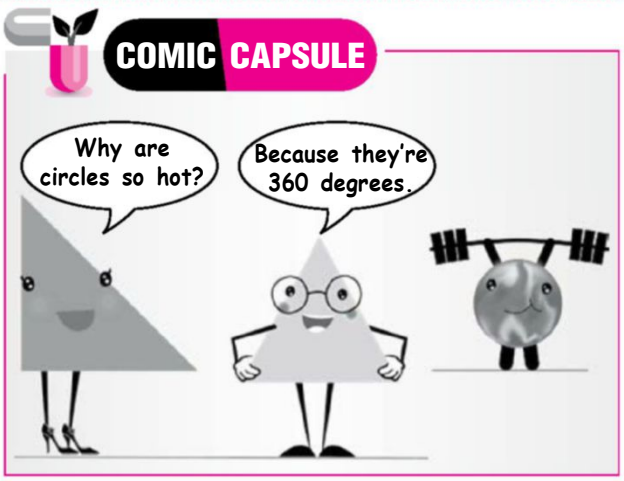
1. (b) : As $P(x)$ divides $Q(P(x)) = (P(x))^2 + cP(x) + d$, it follows that $d = 0$.

So, $P(x)\ Q(x) = (P(x))^2 + c \cdot P(x) \Rightarrow P(x) + c = Q(x)$

i.e., $c=\left(c-\frac{1}{2}\right)x-b$ for all $x \Rightarrow c=\frac{1}{2}$ and $b=-c=\frac{-1}{2}$

$\therefore P(x)=x^2+\frac{x}{2}-\frac{1}{2}$, $Q(x)=x^2+\frac{x}{2}$

and so, $P(Q(x))=\frac{1}{4}(4x^4+4x^3+3x^2+x-2)$ which has only two real roots -1 and $\frac{1}{2}$.



2. (a) : From both the graphs, it is obvious that $y = |\sin x|$ and $y = kx$ intersects at $\alpha = 0$, $\beta \in (0, \pi)$ and $\gamma \in \left(\pi, \frac{3\pi}{2}\right)$. So, at $(\gamma, -\sin\gamma)$ we have,

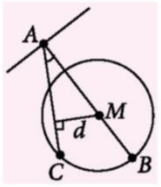
$$\left.\frac{dy}{dx}\right|_{x=\gamma} = -\cos\gamma = \frac{-\sin\gamma - 0}{\gamma - 0} \Rightarrow \tan\gamma = \gamma$$

Hence, $\frac{\cos\gamma}{\sin\gamma + \sin 3\gamma} = \frac{1+\tan^2\gamma}{4\tan\gamma} = \frac{1+\gamma^2}{4\gamma}$

3. (d) : The sequence written to some terms, looks like 0, 1, 1, 2, 0, 2, 2, 1, 0, 1, 1, 2, ... *i.e.*, $a_n = a_{n+8}$

Hence, $\sum_{k=2017}^{2024} a_k = \sum_{k=1}^{8} a_k = 9$

4. (b) : Let $A(t, 9 - t)$



Then, $d = AM\sin 45°$

$= \frac{1}{\sqrt{2}} \cdot \sqrt{(t-2)^2 + (9-t-2)^2}$

Now, $\frac{1}{\sqrt{2}}\sqrt{(t-2)^2 + (9-t-2)^2} \le \sqrt{\frac{17}{2}}$ [$\because$ $d \le$ radius]

On solving we get, $3 \le t \le 6$

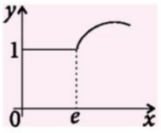
5. (d) : From condition (iii), $a > 0$ and from condition (i), $x = -1$ is axis of symmetry. So, let $f(x) = a(x + 1)^2$. Also, from (i), $f(1) \ge 1$ but from (ii), $f(1) \le 1$

$\Rightarrow \quad f(1) = 1$ So, $a = \frac{1}{4}$. Hence, $f(x) = \frac{1}{4}(x+1)^2$

6. (a) : Notice that $\int_0^{\theta} \sqrt{1+\cos^2 t}\,dt$ is the length of the curve $y = \sin x$ from $(0, 0)$ to $(\theta, \sin\theta)$ and $\sqrt{\theta^2 + \sin^2\theta}$ is the distance between these points.

7. (b) : $f(x) = \lim_{m\to\infty} \frac{m + \log\left(1 + \left(\frac{x}{e}\right)^m\right)}{m}$

$= \begin{cases} 1 \text{ if } 0 \le x \le e \\ \log x \text{ if } x > e \end{cases}$



Hence $f(x)$ is non-differentiable at $x = e$.

8. (a) : Let $d(p)$ denote the distance of a point P from Π, with P being on the same side of Π as A, B, C. Let G_1 be the fixed centroid of ΔABC and G_1' be the centroid of $\Delta A'B'C'$. It is obvious that G is the midpoint of $G_1\,G_1'$. Hence varying G_1' across Π, we have the locus of G as the plane Π' parallel to Π such that $P \in \Pi'$,

$$d(P) = \frac{d(G_1)}{2} = \frac{d(A) + d(B) + d(C)}{6}$$

9. (a) : Let $\frac{z_1}{z_2} = e^{i\theta}, \frac{z_2}{z_3} = e^{i\alpha}$, then $\frac{z_3}{z_1} = e^{-i(\theta+\alpha)}$

$$\frac{z_1}{z_2} + \frac{z_2}{z_3} + \frac{z_3}{z_1} = 1 \Rightarrow \text{Im}\left(\frac{z_1}{z_2}\right) + \text{Im}\left(\frac{z_2}{z_3}\right) + \text{Im}\left(\frac{z_3}{z_1}\right) = 0$$

i.e., $\sin\theta + \sin\alpha - \sin(\theta + \alpha) = 0$

Simplifying, $4\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\theta+\alpha}{2}\right) = 0$

It follows that $\theta = 2k\pi$ or $\alpha = 2k\pi$ or $\theta + \alpha = 2k\pi$, $(k \in I)$.

i.e., $z_1 = z_2$, $z_2 = z_3$ or $z_3 = z_1$.

If $z_1 = z_2$, then $\frac{z_1}{z_3} + \frac{z_3}{z_1} = 0 \Rightarrow \frac{z_3}{z_1} = \pm i$

$\Rightarrow |az_1 + bz_2 + cz_3| = |z_1||a + b \pm ci| = \sqrt{(a+b)^2 + c^2}$

Other possible values are $\sqrt{(b+c)^2 + a^2}$ and $\sqrt{(c+a)^2 + b^2}$.

10. (b) : Notice that $2I_2 = \int_{f(a)}^{f(b)} 2x \cdot [b - f^{-1}(x)]dx$

$= \int_a^b (b-y)\cdot 2f(y)f'(y)dy$ [$\because$ $y = f^{-1}(x)$]

$= (b-y)f^2(y)\Big|_a^b + \int_a^b f^2(y)dy$ [Integrating by parts]

$= \int_a^b [f^2(y) - f^2(a)]dy = \int_a^b [f^2(x) - f^2(a)]dx = I_1$

11. (b) **12. (b)**

Concept : Instead of $f(x)$, we analyse its inverse

$$f^{-1}(x) = g(y) = \frac{1}{26}\left(\frac{27 - y^3}{y}\right) \text{ in } [1, 3]$$

Notice that $g'(y) < 0$ for $y > 0$ and hence, so is $f(x)$.

and $f'(x) = \frac{1}{g'(f(x))} = \frac{-26(f(x))^2}{27 + 2(f(x))^3}$

So, given integral becomes

$$I = \int_1^3 \frac{1}{26}\cdot(27 + 2y^3)\log y\,dy = \frac{1}{52}[243\log 3 - 128]$$

13. (a) **14. (a)**

A can win after the $(n+1)^{\text{th}}$ game in the following 2 ways.

(i) A wins exactly one of the first n games, draws $(n-1)$ games and wins the $(n+1)^{\text{th}}$ game.

(ii) A loses exactly one of the first n games, wins exactly one of the first n games and draws $(n-2)$ games and

finally wins the $(n+1)^{th}$ game.

For (i), probability = $({}^nP_1 \cdot ab^{n-1})a$ and for (ii) the probability is $({}^nP_2)\cdot ac\cdot b^{n-2}\cdot a$

So, required probability

$= na^2b^{n-1} + n(n-1)a^2cb^{n-2}$

$= na^2[b^{n-1} + (n-1)\cdot c\cdot b^{n-2}]$

Now, for B winning, we put $n = 3$ and interchange a and c in the above equation.

So required probability $= 3c^2[b^2 + 2ab] = 3bc^2[b + 2a]$

15. (3) : Characteristic equation of A is

$|A - \lambda I| = 0$ *i.e.*, $\lambda^2 + 3\lambda + 2 = 0$

So, $(A+I)^3 = A + I$

$\Rightarrow A = (A+I)^3 + (-I)^3$

i.e., $B = A + I$ and $C = -I$

$\Rightarrow Tr(B) + Tr(C) = (-1) + (-2) = -3$

$\therefore |Tr(B) + Tr(C)| = 3$

16. (4) : Let focus $S(a, 0)$, $A(at_1^2, 2at_1)$ etc.

$$SA \perp BC \Rightarrow \frac{2t_1}{t_1^2-1}\cdot\frac{2}{t_3+t_2} = -1 \qquad ...(1)$$

$$\Rightarrow 4t_1 + t_1^2(t_2+t_3) = t_2 + t_3$$

Similarly, $4t_2 + t_2^2(t_3+t_1) = t_3 + t_1$

Simplifying, $\Sigma t_1t_2 = -5$ ...(2)

Using (1) and (2), $t_1 + t_2 + t_3 + t_1t_2t_3 = 0$

Hence, $\Pi(t_1 - 1) = t_1t_2t_3 - \Sigma t_1t_2 + \Sigma t_1 - 1 = 4$

17. (2) : Let $A = (x, 0)$, $B = \left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$, $C = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Then the given expression is $AB - AC$

So, from triangle inequality, $|AB - AC| < BC$

and $BC = 1$. So, given expression $\in (-1, 1)$.

Hence, $|b - a| = 2$

18. (0) : Let $f(x) = \sin(\sin x)$ and $g(x) = \cos(\cos x)$

Then $g(x) - f(x)$

$$= 2\cos\left(\frac{\pi}{4} - \frac{\cos x - \sin x}{2}\right)\cdot\sin\left(\frac{\pi}{4} - \frac{\cos x + \sin x}{2}\right)$$

Now, $\left|\frac{\cos x \pm \sin x}{2}\right| = \frac{\sqrt{2}}{2}\left|\sin\left(x \pm \frac{\pi}{4}\right)\right| < \frac{\pi}{4}$

$\Rightarrow g(x) - f(x) > 0$ and $f(x)$ is increasing

So, $f(f(x)) < f(g(x)) < g(g(x))$

Hence, no solution.

19. (0) : $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{e^{x^2/2}\cdot f(x)}{e^{x^2/2}}$

Now applying L-Hospital's Rule twice, we get

$$\lim_{x\to\infty} \frac{f''(x) + 2xf'(x) + (x^2+1)f(x)}{x^2+1} = 0$$

20. (9) : Let $a = 1 + x + x^2 + ... + x^{n-1}$

and $b = 1 + 3x + 5x^2 + ... + (2n-1)x^{n-1}$

Then $I_n = \int_0^1 (ab)\,dx$

So, $I_{n+1} = \int_0^1 (a + x^n)(b + (2n+1)x^n)dx$

$$= I_n + (2n+1)\int_0^1 ax^n dx + \int_0^1 bx^n dx + \int_0^1 (2n+1)\,x^{2n}dx$$

On simplification, $I_{n+1} = I_n + 1 + 2n$

So, $I_9 = I_8 + 1 + 2(8)$

$I_8 = I_7 + 1 + 2(7)$

.....

.....

$I_2 = I_1 + 1 + 2(1)$

Adding all we get, $I_9 = I_1 + 80$ and $I_1 = \int_0^1 (1)(1)dx = 1$

So, $I_9 = 81 \Rightarrow \sqrt{I_9} = 9$

Form IV

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I, Mahabir Singh, authorised signatory for MTG Learning Media Pvt. Ltd. hereby declare that particulars given above are true to the best of my knowledge and belief.

For MTG Learning Media Pvt. Ltd.
Mahabir Singh
Director

CONCEPT MAP

SEQUENCES AND SERIES

PROBABILITY

CONCEPT MAP

Class XI **Class XII**

SERIES

If $a_1, a_2, ..., a_n$ is a sequence, then the expansion $a_1 + a_2 + ... + a_n + ...$ is called the series.

SEQUENCE

A sequence is a function from natural number N (domain) to real numbers (codomain)

Sum of *n* terms of special series

- Sum of n natural numbers $\sum n = \frac{n(n+1)}{2}$
- Sum of squares of n natural numbers $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$
- Sum of cubes of n natural numbers

$$\sum n^3 = \frac{n^2(n+1)^2}{4} = \left(\sum n\right)^2$$

Basic Properties

- If a constant is added / subtracted / multiplied / divided to each term of an A.P., then the resulting sequence is also an A.P.
- Selection of terms in an A.P.
  - Any three numbers in A.P. can be taken as $a - d, a, a + d$.
  - Any four numbers in A.P. can be taken as $a - 3d, a - d, a + d, a + 3d$.
- If each term of a G.P. is multiplied / divided by a same non-zero number, then the resulting sequence is also in G.P.
- Selection of terms in a G.P.
  - Any three numbers in G.P. can be taken as $a/r, a, ar$.
  - Any four numbers in G.P. can be taken as $a/r^3, a/r, ar, ar^3$.

Arithmetic Mean (A.M.)

- For two numbers a and b, A.M. is $a+b/2$.
- $A_k = a + k\left(\frac{b-a}{n+1}\right), \forall\ k = 1, 2, ..., n$

where $A_1, ..., A_n$ are n arithmetic means inserted between two numbers a and b.

Geometric Mean (G.M.)

- For two numbers a and b, G.M. is $\sqrt{ab}$.
- $G_k = a\left(\frac{b}{a}\right)^{k/n+1} \quad \forall\ k = 1, 2, 3, ..., n$

where $G_1, ..., G_n$ are n geometric means inserted between two numbers a and b.

Harmonic Mean (H.M.)

- For two numbers a and b, H.M. is $2ab/a + b$.
- $H_n = \frac{(n+1)ab}{an+b} \quad \forall\ n = 1, 2, ...$

where a and b are two numbers and $H_1, H_2, ..., H_n$ are n harmonic means inserted between them.

Progression

If the terms of a sequence are written under specific conditions, then the sequence is called progression.

Types

Arithmetic Progression (A.P.)

A sequence whose terms increases or decreases by a fixed number.

- **n^{th} term :** $T_n = a + (n - 1)d$ where d (common difference) $= T_n - T_{n-1}$, a = first term,
- **n^{th} term from end :** $T_n' = l - (n - 1)d$, where l = last term
- **Sum of n terms :** $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + l]$

Geometric Progression (G.P.)

A sequence of non-zero numbers for which the ratio of a term to its just preceding term is always constant.

- **n^{th} term :** $T_n = ar^{n-1}$, where r (common ratio) $= T_n/T_{n-1}$, a = first term
- **n^{th} term from end :** $T_n' = l/r^{n-1}$, l = last term
- **Sum of n terms :** $S_n = \begin{cases} \frac{a(r^n-1)}{r-1}, & r > 1 \\ \frac{a(1-r^n)}{1-r}, & r < 1 \\ an, & r = 1 \end{cases}$; $S_\infty = \frac{a}{1-r}, |r| < 1$

Harmonic Progression (H.P.)

A sequence $a_1, a_2, a_3, ...$ in which reciprocal of terms form an A.P.

- **n^{th} term:** $T_n = \frac{1}{\frac{1}{a_1} + (n-1)\left(\frac{1}{a_2} - \frac{1}{a_1}\right)} = \frac{a_1 a_2}{a_2 + (n-1)(a_1 - a_2)}$

Note : No term of an H.P. can be zero.

Multiplication

- $P(A \cap B) = P(A) \cdot P(B \mid A) = P(B) \cdot P(A \mid B)$, provided $P(A) \neq 0$, $P(B) \neq 0$ *i.e.* $P(AB) = P(A) \cdot P(B \mid A) = P(B) \cdot P(A \mid B)$, provided $P(A) \neq 0$, $P(B) \neq 0$

Total Probability

- $P(A) = P(E_1)\,P(A \mid E_1) + P(E_2)\,P(A \mid E_2) + ... + P(E_n)\,P(A \mid E_n)$ where, $E_1, E_2, E_3, ..., E_n$ are mutually exclusive and exhaustive events.

Bayes'

- $P(E_i|A) = \frac{P(E_i)\,P(A|E_i)}{\Sigma P(E_i)\,P(A|E_i)}$, where i = 1, 2, 3, ..., n, where, $E_1, E_2, E_3, ..., E_n$ are mutually exclusive and exhaustive events.

Theorems

Independent Events

Occurrence or non-occurrence of one does not affect the occurrence of the other *i.e.*,
$P(A \mid B) = P(A)$ provided $P(B) \neq 0$,
$P(B \mid A) = P(B)$ provided $P(A) \neq 0$
[For n events, $P(A_1 \cap A_2 \cap ..... \cap A_n) = P(A_1) \cdot P(A_2) ..... P(A_n)$]

Random Variables and its Probability Distributions

- A real valued function, whose domain is the sample space of a random experiment. Generally, it is denoted by X.
- Let real numbers $x_1, x_2, ..., x_n$ are the possible values of a random variable X and $p_1, p_2, ..., p_n$ are the corresponding probabilities to each value of the random variable X. Then the probability distribution is

X :	x_1	x_2	...	x_n
$P(X)$:	p_1	p_2	...	p_n.

- $P(x_k)$ *i.e.*, $P(X = x_k)$ lies between 0 and 1 for $k = 1, 2, \ldots, n$.
- $\sum_{i=1}^{n} p_i = 1$

Mean

- $\bar{X} = E(X) = \mu = p_1x_1 + p_2x_2 + ... + p_nx_n = \sum_{i=1}^{n} p_i x_i$

PROBABILITY

Conditional Probability

- Probability of occurrence of an event A, given that B has already occurred. *i.e.*, $P(A|B) = P(A \cap B) / P(B)$

Properties

- $0 \leq P(A|B) \leq 1$
- $P((A \cup B)|F) = P(A|F) + P(B|F) - P((A \cap B)|F)$, where $P(F) \neq 0$ $= P(A|F) + P(B|F)$ (if A & B are disjoint)
- $P(A'|B) = 1 - P(A|B)$

Variance & Standard Deviation

- $\text{Var}(X) = \sigma^2 = \sum_{i=1}^{n} p_i x_i^2 - \left(\sum_{i=1}^{n} p_i x_i\right)^2$

or $\text{Var}(X) = E(X^2) - \{E(X)\}^2$

- Standard deviation $(\sigma) = \sqrt{\text{Var}(X)}$

Bernoulli's Trials

Trials of a random experiment are called bernoulli's trials if

- they are finite in number.
- they are independent of each other.
- each trial has exactly two outcomes *i.e.*, success or failure.
- the probability of success or failure remains same in each trial.

Binomial Distribution

$P(X = r) = {}^nC_r\, p^r q^{n-r}$, where $r = 0, 1, 2, ...n$, where p and q are probabilities of success and failure respectively.

Mean

- $\bar{X} = np$

Variance

- $\sigma^2 = npq$

Standard Deviation

- $\sigma = \sqrt{npq}$

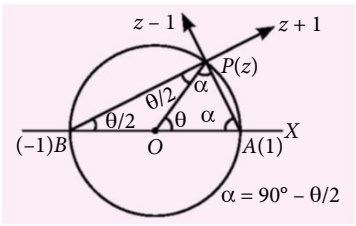
YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. If z is a non-real complex number lying on the circle $|z| = 1$, then show that $\tan\left(\frac{\arg(z)}{2}\right) = \frac{i(1-z)}{1+z}$.

(Pratiksha, U.P.)

Ans. The circle $|z| = 1$ meets the real axis at points A and B whose complex representations are 1 and –1 respectively.



If P whose complex representation is z is any point on the given circle and $\theta = \arg(z)$, then

$$PB = 2\cos\left(\frac{\theta}{2}\right) \text{ and } PA = 2\sin\left(\frac{\theta}{2}\right)$$

The complex representations of BP and AP are $z + 1$ and $z - 1$ respectively.

$$\therefore \quad \frac{z-1}{z+1} = \frac{AP}{BP}e^{i(\pi/2)} = i\tan\left(\frac{\theta}{2}\right) \qquad [\because AP \perp BP]$$

$$\Rightarrow \quad \tan\left(\frac{\theta}{2}\right) = \frac{z-1}{i(z+1)} = \frac{i(1-z)}{z+1}$$

$$\Rightarrow \quad \tan\left(\frac{\arg(z)}{2}\right) = \frac{i(1-z)}{1+z}$$

2. Let X be a set containing n elements. If two subsets A and B of X are chosen at random, find the probability that A and B have same number of elements.

(Sidharth, Mumbai)

Ans. While forming a subset, for each element of X, we have two choices (selecting or not selecting that particular element). Thus, the number of subsets that can be formed of X, is 2^n.

The sample space S = {choosing two subsets of X}.

Then, $n(S) = 2^n \cdot 2^n = 2^{2n}$

Now, the number of subsets of X which contain exactly r elements is ${}^nC_r$. Thus, two subsets both containing r elements can be chosen in ${}^nC_r \cdot {}^nC_r = ({}^nC_r)^2$ ways.

If E = {choosing two subsets both containing same number of elements}, then

$$n(E) = ({}^nC_0)^2 + ({}^nC_1)^2 + \ldots\ldots + ({}^nC_n)^2$$

$$= {}^{2n}C_n = \frac{1\cdot 2\cdot 3\cdot 4\ldots\ldots(2n-1)(2n)}{n!n!}$$

$$= \frac{[1\cdot 3\cdot 5\ldots\ldots(2n-1)][2\cdot 4\cdot 6\ldots\ldots(2n)]}{(n!)^2}$$

$$= \frac{[1\cdot 3\cdot 5\ldots\ldots(2n-1)]\,2^n\,[1\cdot 2\cdot 3\ldots\ldots n]}{(n!)^2}$$

$$= \frac{2^n[1\cdot 3\cdot 5\ldots\ldots(2n-1)]}{n!}$$

Hence, the required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{1\cdot 3\cdot 5\ldots\ldots(2n-1)}{2^n\,(n!)}$$

3. In any ΔABC, prove that

(a) $ca\cos B - bc\cos A = a^2 - b^2$.

(b) $2(bc\cos A + ca\cos B + ab\cos C) = a^2 + b^2 + c^2$.

(Manish Kumar, Bihar)

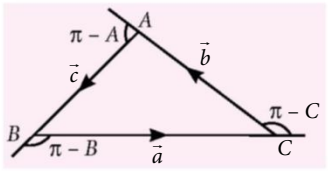
Ans. Let $BC = \vec{a}$, $CA = \vec{b}$, $AB = \vec{c}$

Then, $\vec{a} + \vec{b} + \vec{c} = 0$ ...(i)

Now, $\vec{a}\cdot\vec{b} = ab\cos(\pi - C) = -ab\cos C$ ...(ii)

$\vec{b}\cdot\vec{c} = bc\cos(\pi - A) = -bc\cos A$ ...(iii)

$\vec{c}\cdot\vec{a} = ca\cos(\pi - B) = -ca\cos B$ ...(iv)



(a) Subtracting (iv) from (iii), we have

$ca\cos B - bc\cos A = \vec{b}\cdot\vec{c} - \vec{c}\cdot\vec{a} = \vec{c}\cdot(\vec{b} - \vec{a})$

$= -(\vec{a} + \vec{b})\cdot(\vec{b} - \vec{a})$ [Using (i)]

$= |\vec{a}|^2 - |\vec{b}|^2 = a^2 - b^2$.

(b) From (ii), (iii) and (iv), we have

$2(ab\cos C + bc\cos A + ca\cos B)$

$= -2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$= -[(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c}) + (\vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) + (\vec{c}\cdot\vec{a} + \vec{a}\cdot\vec{b})]$

$= -[\vec{b}\cdot(\vec{a} + \vec{c}) + \vec{c}\cdot(\vec{a} + \vec{b}) + \vec{a}\cdot(\vec{b} + \vec{c})]$

$= -[\vec{b}\cdot(-\vec{b}) + \vec{c}\cdot(-\vec{c}) + \vec{a}\cdot(-\vec{a})]$ [Using (i)]

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 = a^2 + b^2 + c^2$. ◆◆

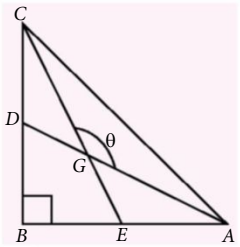
MATHS MUSING

SOLUTION SET-194

1. (d) : Let $AB = BC = \lambda$,

$\therefore$ $AC = \lambda\sqrt{2}$ and $BE = \dfrac{\lambda}{2}$

$\because$ AD and CE are medians, so G is centroid.



Then, $GA = GC = \dfrac{2}{3}\ CE$

$$= \frac{2}{3}\sqrt{(BC)^2 + (BE)^2} = \frac{2}{3}\sqrt{\lambda^2 + \left(\frac{\lambda}{2}\right)^2} = \frac{2}{3}\cdot\frac{\lambda}{2}\sqrt{5} = \frac{\sqrt{5}\lambda}{3}$$

$$\therefore \quad \cos\theta = \frac{(GA)^2 + (GC)^2 - (AC)^2}{2GA \cdot GC}$$

$$= \frac{\frac{5}{9}\lambda^2 + \frac{5}{9}\lambda^2 - 2\lambda^2}{2\cdot\frac{\lambda\sqrt{5}}{3}\cdot\frac{\lambda\sqrt{5}}{3}} = \frac{-8\lambda^2}{10\lambda^2} = -\frac{4}{5}$$

2. (b) : Centre, $C \equiv (7, 5)$ and radius $r = \sqrt{49 + 25 + 151} = 15$

If $P = (2, -7)$, then shortest distance $= |CP - r|$

$= |\sqrt{25 + 144} - 15| = |13 - 15| = 2$

3. (a) : $f'(x) = ax^2 + 1 + \cos x > 0$, for $a > 0$ and $\forall\ x \in R$

$\Rightarrow$ $f(x)$ is an increasing function

$\Rightarrow$ $\max f(x) = f(3)$ and $\min f(x) = f(2)$

$$\Rightarrow \text{ Difference} = f(3) - f(2) = \int_2^3 (at^2 + 1 + \cos t)\,dt$$

$$= \frac{19}{3}a + 1 + (\sin 3 - \sin 2)$$

4. (a) : For conjugate diameters $m_1 m_2 = -\dfrac{b^2}{a^2}$

Here, $2x^2 + xy - 3y^2 = 0$

$\Rightarrow$ $(x - y)(2x + 3y) = 0$

Thus, slopes of conjugate diameters are $1, -\dfrac{2}{3}$.

$\therefore$ $\dfrac{b^2}{a^2} = 2/3$ and $e = \sqrt{(1 - b^2/a^2)} = \dfrac{1}{\sqrt{3}}$

5. (b) : Here, $A = \displaystyle\int_0^{\pi} \frac{\cos x}{(x+2)^2}\,dx = \left[\cos x \cdot \left(-\frac{1}{x+2}\right)\right]_0^{\pi}$

$$-\int_0^{\pi} (-\sin x)\left(-\frac{1}{x+2}\right)dx$$

$$= \frac{1}{\pi+2} + \frac{1}{2} - \int_0^{\pi} \frac{\sin x}{x+2}\,dx$$

$$\Rightarrow \quad A = \frac{1}{\pi+2} + \frac{1}{2} - 2\int_0^{\pi/2} \frac{\sin 2x}{2x+2}\,dx$$

$$\Rightarrow \quad \int_0^{\pi/2} \frac{\sin 2x}{x+1}\,dx = \frac{1}{\pi+2} + \frac{1}{2} - A$$

6. (a, b, c, d) : In an equilateral triangle, orthocentre, circumcentre and centroid coincide.

For (c), recall for an equilateral triangle

$(z_1 + z_2 + z_3)^2 = 3(z_2z_3 + z_3z_1 + z_1z_2)$

7. (b) : Here, $(foh)(x) = f(h(x)) = f(\tan x)$

$$= \frac{\tan^2 x - \tan x + 1}{\tan^2 x + \tan x + 1}, -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$= \frac{t^2 - t + 1}{t^2 + t + 1}, t = \tan x \in R \quad \therefore \text{ Range of } foh \text{ is } \left[\frac{1}{3},\ 3\right]$$

8. (d) : Here, $(goh)(x) = g(h(x)) = g(\tan x) = e^{\tan x} = e^t$, $t = \tan x \in R$

$\therefore$ Range of goh is $(0, \infty)$. $(\because$ Range of e^x is $(0, \infty))$

9. (146) : $\overrightarrow{AB} = (2\hat{i} + 3\hat{j} - 4\hat{k}) - (3\hat{i} - 2\hat{j} - \hat{k}) = -\hat{i} + 5\hat{j} - 3\hat{k}$

$\overrightarrow{AC} = (-\hat{i} + \hat{j} + 2\hat{k}) - (3\hat{i} - 2\hat{j} - \hat{k}) = -4\hat{i} + 3\hat{j} + 3\hat{k}$

$\overrightarrow{AD} = (4\hat{i} + 5\hat{j} + \lambda\hat{k}) - (3\hat{i} - 2\hat{j} - \hat{k}) = \hat{i} + 7\hat{j} + (\lambda + 1)\hat{k}$

Vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ will be coplanar

$$\Rightarrow \begin{vmatrix} -1 & 5 & -3 \\ -4 & 3 & 3 \\ 1 & 7 & \lambda + 1 \end{vmatrix} = 0 \Rightarrow 17\lambda + 146 = 0 \Rightarrow -17\lambda = 146$$

10. (b) : **(P) $\rightarrow$ (1, 2), (Q) $\rightarrow$ (3, 4), (R) $\rightarrow$ (1, 3, 4), (S) $\rightarrow$ (2, 3, 4)**

(P) $3\sin^2 x - \sin x \cos x - 4\cos^2 x = 0$ (if $\cos x \neq 0$)

$\Rightarrow$ $3\tan^2 x - \tan x - 4 = 0$ $\Rightarrow$ $(3\tan x - 4)(\tan x + 1) = 0$

$\Rightarrow$ $\tan x = 4/3$ or $\tan x = -1$

(Q) $2\sin^2\left(\dfrac{\pi}{2}\cos^2 x\right) = 2\sin^2\left(\dfrac{\pi}{2}\sin 2x\right)$

$\Rightarrow$ $\cos^2 x = \sin 2x \Rightarrow \cos x(\cos x - 2\sin x) = 0$

$\Rightarrow$ $\cos x = 0$ or $\tan x = 1/2 \Rightarrow \cos 2x = 3/5$

(R) $6(\tan^2 x + 1) - 11\tan x - 2 = 0$

$\Rightarrow$ $6\tan^2 x - 11\tan x + 4 = 0$

$\Rightarrow$ $(3\tan x - 4)(2\tan x - 1) = 0$

$\Rightarrow$ $\tan x = 4/3$ or $\tan x = 1/2 \Rightarrow \cos 2x = 3/5$

(S) $1 - 2\sin^2 x - \sin x \cos x = \sin^2 x$

$\Rightarrow$ $3\sin^2 x + \sin x \cos x - 1 = 0$

$\Rightarrow$ $2\sin^2 x + \sin\cos x - \cos^2 x = 0$

$\Rightarrow$ $2\tan^2 x + \tan x - 1 = 0$ (if $\cos x \neq 0$)

$\Rightarrow$ $(2\tan x - 1)(\tan x + 1) = 0$

$\Rightarrow$ $\tan x = -1$ or $\tan x = 1/2 \Rightarrow \cos 2x = 3/5$

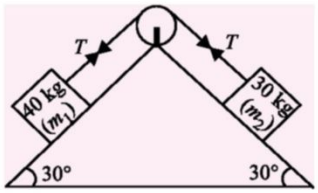
FULL LENGTH PRACTICE PAPER

BITSAT

Exam date: 16th to 26th May 2019

SECTION-I (PHYSICS)

1. Two masses 40 kg and 30 kg are connected by a weightless string passing over a frictionless pulley as shown in the figure.



The tension in the string will be

(a) 188 N (b) 368 N (c) 288 N (d) 168 N

2. The half-life of ^{60}Co is approximately 5.25 years. In a sample containing 1 g of freshly prepared ^{60}Co, how much of the isotope will be left after 21 years?

(a) 125 mg (b) 62.5 mg
(c) 31.25 mg (d) nothing will be left

3. At constant temperature, the volume of a gas is to be decreased by 4%. The pressure must be increased by

(a) 5.34% (b) 4.16% (c) 2.96% (d) 3.86%

4. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is 54 μT. What will be its value at the centre of the loop?

(a) 250 μT (b) 150 μT (c) 125 μT (d) 75 μT

5. The input resistance of a silicon transistor is 100 Ω. Base current is changed by 40 μA which results in a change in collector currently by 2 mA. This transistor is used as a common emitter amplifier with a load resistance of 4 kΩ. The voltage gain of the amplifier is

(a) 2000 (b) 3000 (c) 4000 (d) 1000

6. An alternating emf $e = 220\sqrt{2}\sin 100t$ V is applied to a capacitor 1 μF. The current flowing through the capacitor is

(a) 22 mA (b) 12 mA (c) 32 mA (d) 42 mA

7. In a race for 100 m dash, the first and the second runners have a gap of one metre at the mid way stage. Assuming the first runner goes steady, by what percentage should the second runner increase his speed just to win the race?

(a) 2% (b) 4%
(c) more than 4% (d) less than 4%

8. Two linear SHMs of equal amplitude A and angular frequencies ω and 2ω are impressed on a particle along the axes x and y respectively. If the initial phase difference between them is π/2, the resultant path followed by the particle is

(a) $y^2 = x^2(1 - x^2/A^2)$ (b) $y^2 = 2x^2(1 - x^2/A^2)$
(c) $y^2 = 4x^2(1 - x^2/A^2)$ (d) $y^2 = 8x^2(1 - x^2/A^2)$

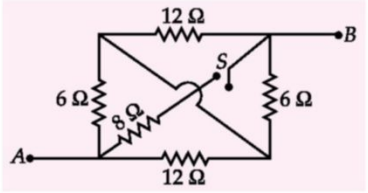
9. A calorie is a unit of heat and equals 4.2 J. Suppose we employ a system of units in which the unit of mass is α kg, the unit of length is β m and the unit of time is γ s. In this new system, 1 calorie will be

(a) $\alpha^{-1}\beta^{-2}\gamma^2$ (b) $4.2\alpha\beta^2\gamma^2$
(c) $\alpha\beta^2\gamma^2$ (d) $4.2\alpha^{-1}\beta^{-2}\gamma^2$

10. According to Bohr's theory the energy of an electron in hydrogen atom starts corresponding to n = 1, 2, 3, 4 is – 13.6 eV, – 3.4 eV, – 1.51 eV and –0.85 eV respectively. In order to obtain emission of H_β line of Balmer series, the energy that has to be given to a normal hydrogen atom is

(a) 12.75 eV (b) 14.45 eV
(c) 2.55 eV (d) 4.25 eV

11. The equivalent resistance between points A and B with switch S open and closed are respectively



(a) 4 Ω, 8 Ω (b) 8 Ω, 4 Ω
(c) 6 Ω, 9 Ω (d) 9 Ω, 6 Ω

12. A sand bag of mass m is suspended from a long string. A bullet of mass $\frac{m}{20}$ moving with a horizontal velocity v strikes it and gets embedded into it. Calculate the velocity gained by the bag in this process and fraction of energy lost in the process.

(a) $\frac{v}{21}, \frac{20}{21}$ (b) $\frac{v}{24}, \frac{18}{20}$ (c) $\frac{v}{21}, \frac{20}{22}$ (d) $\frac{v}{19}, \frac{20}{21}$

13. Consider an *n-p-n* transistor amplifier in common emitter configuration. The current gain of the transistor is 100. If the collector current changes by 1 mA, what will be the change in emitter current ?

(a) 1.1 mA (b) 1.01 mA (c) 0.01 mA (d) 10 mA

14. When radiation of the wavelength λ is incident on a metallic surface, the stopping potential is 4.8 V. If the same surface is illuminated with radiation of double the wavelength, then the stopping potential becomes 1.6 V. Then the threshold wavelength for the surface is

(a) 2λ (b) 4λ (c) 6λ (d) 8λ

15. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectrons emission from this substance is approximately

(a) 540 nm (b) 400 nm (c) 310 nm (d) 220 nm

16. The potential difference between the plates of a parallel plate capacitor is changing at the rate of 10^6 V s^{-1}. If the capacitance is 2 μF, the displacement current in the dielectric of the capacitor will be

(a) 1 A (b) 2 A (c) 3 A (d) 4 A

17. In an experiment with sonometer, a tuning fork of frequency 256 Hz resonates with a length of 25 cm and another tuning fork resonates with a length of 16 cm. Tension in the string remains constant, the frequency of the second tuning fork is

(a) 204 Hz (b) 160 Hz (c) 400 Hz (d) 320 Hz

18. The maximum range of projectile fired with some initial velocity is found to be 1000 m, in the absence of wind and air resistance. The maximum height reached by the projectile is

(a) 250 m (b) 500 m (c) 1000 m (d) 2000 m

19. A physical quantity X is given by $X = \frac{2k^3l^2}{m\sqrt{n}}$

The percentage error in the measurements of k, l, m and n are 1%, 2%, 3% and 4% respectively. The value of X is uncertain by

(a) 8% (b) 10%
(c) 12% (d) none of these

20. A conducting metal circular-wire-loop of radius r is placed perpendicular to a magnetic field which varies with time as $B = B_0 e^{-\frac{t}{\tau}}$, where B_0 and τ are constants, at time $t = 0$. If the resistance of the loop is R then the heat generated in the loop after a long time ($t \to \infty$) is

(a) $\frac{\pi^2 r^4 B_0^4}{2\tau R}$ (b) $\frac{\pi^2 r^4 B_0^2}{2\tau R}$
(c) $\frac{\pi^2 r^4 B_0^2 R}{\tau}$ (d) $\frac{\pi^2 r^4 B_0^2}{\tau R}$

21. The cylindrical tube of spray pump has a cross-section of 8 cm^2, one end of which has 40 fine holes each of area 10^{-8} m^2. If the liquid flows inside the tube with a speed of 0.15 m min^{-1}, the speed with which the liquid is ejected through the holes is

(a) 50 m s^{-1} (b) 5 m s^{-1}
(c) 0.05 m s^{-1} (d) 0.5 m s^{-1}

22. A body is fired vertically upwards. At half the maximum height, the velocity of the body is 10 m s^{-1}. The maximum height raised by the body is (Take g = 10 m s^{-2})

(a) 5 m (b) 10 m (c) 15 m (d) 20 m

23. A satellite in a circular orbit of radius R has a period of 4 hours. Another satellite with orbital radius $3R$ around the same planet will have a period (in hours)

(a) 16 (b) 4 (c) $4\sqrt{27}$ (d) $4\sqrt{8}$

24. A body of mass M suspended from two springs separately executes simple harmonic motion. During oscillation the maximum velocity is equal in both cases. The ratio of amplitude $\frac{A_1}{A_2}$ is

(a) $\frac{k_1}{k_2}$ (b) $\frac{k_2}{k_1}$ (c) $\sqrt{\frac{k_2}{k_1}}$ (d) $\frac{k_1^2}{k_2^2}$

25. Three point charges $+q$, $-2q$ and $+q$ are placed at points ($x = 0, y = a, z = 0$), ($x = 0, y = 0, z = 0$) and ($x = a, y = 0, z = 0$) respectively. The magnitude and direction of the electric dipole moment vector of this charge assembly are

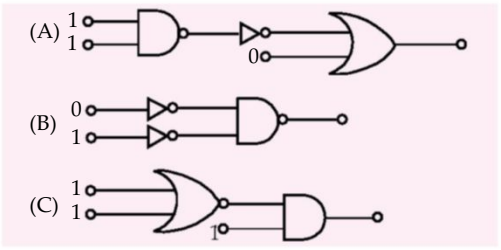
(a) $\sqrt{2}\,qa$ along the line joining the points ($x = 0, y = 0, z = 0$) and ($x = a, y = a, z = 0$).

(b) qa along the line joining the points $(x = 0, y = 0, z = 0)$ and $(x = a, y = a, z = 0)$.

(c) $\sqrt{2}\,qa$ along $+x$ direction

(d) $\sqrt{2}\,qa$ along $+y$ direction.

26. In the following combinations of logic gates, the outputs of A, B and C are respectively



(a) 0, 1, 1 (b) 0, 1, 0 (c) 1, 1, 0 (d) 1, 0, 1

27. A wire of length 100 cm is connected to a cell of emf 2 V and negligible internal resistance. The resistance of the wire is 3 Ω. The additional resistance required to produce a potential difference of 1 millivolt per cm is

(a) 60 Ω (b) 47 Ω (c) 57 Ω (d) 35 Ω

28. Two slits separated by a distance of 1 mm are illuminated with red light of wavelength 6.5×10^{-7} m. The interference fringes are observed on a screen placed 1 m from the slits. The distance between the third dark fringe and the fifth bright fringe is equal to

(a) 0.65 mm (b) 1.63 mm
(c) 3.25 mm (d) 4.88 mm

29. A metal rod of Young's modulus Y and coefficient of thermal expansion α is held at its two ends such that its length remains invariant. If its temperature is raised by t °C, the linear stress developed in it is

(a) $\frac{\alpha t}{Y}$ (b) $\frac{Y}{\alpha t}$ (c) $Y\alpha t$ (d) $\frac{1}{(Y\alpha t)}$

30. When a dc voltage of 200 V is applied to a coil of self inductance $\left(2\sqrt{3}/\pi\right)$ H, a current of 1 A flows through it. But by replacing dc source with ac source of 200 V, the current in the coil is reduced to 0.5 A. Then the frequency of ac supply is

(a) 100 Hz (b) 75 Hz (c) 60 Hz (d) 50 Hz

31. A compound microscope has an eye piece of focal length 10 cm and an objective of focal length 4 cm. Find the magnification, if an object is kept at a distance of 5 cm from the objective, so that the final image is formed at the least distance of distinct vision 20 cm.

(a) 12 (b) 11 (c) 10 (d) 13

32. The angular momentum of an electron in the hydrogen atom is $\frac{3h}{2\pi}$. Here, h is Planck's constant. The kinetic energy of this electron is

(a) 4.35 eV (b) 1.51 eV (c) 3.4 eV (d) 6.8 eV

33. If the length of stretched string is shortened by 40% and the tension is increased by 44%, then the ratio of the final and initial fundamental frequencies is

(a) 2 : 1 (b) 3 : 2 (c) 3 : 4 (d) 1 : 3

34. A uniform thin bar of mass $6m$ and length $12L$ is bent to make a regular hexagon. Its moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is

(a) $20mL^2$ (b) $6mL^2$ (c) $\frac{12}{5}mL^2$ (d) $30mL^2$

35. A solenoid of length 30 cm with 10 turns per centimetre and area of cross-section 40 cm^2 completely surrounds another co-axial solenoid of same length, area of cross-section 20 cm^2 with 40 turns per centimeter. The mutual inductance of the system is

(a) 10 H (b) 8 H (c) 3 mH (d) 30 mH

36. An object is displaced from position vector $\vec{r}_1 = (2\hat{i} + 3\hat{j})$ m to $\vec{r}_2 = (4\hat{i} + 6\hat{j})$ m under a force $\vec{F} = (3x^2\hat{i} + 2y\hat{j})$ N. The work done by this force is

(a) 63 J (b) 73 J (c) 83 J (d) 93 J

37. A prism of refractive index μ and angle A is placed in the minimum deviation position. If the angle of minimum deviation is A, then the value of A in terms of μ is

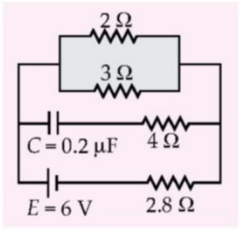
(a) $\sin^{-1}\left(\frac{\mu}{2}\right)$ (b) $\sin^{-1}\left(\sqrt{\frac{\mu-1}{2}}\right)$

(c) $2\cos^{-1}\left(\frac{\mu}{2}\right)$ (d) $\cos^{-1}\left(\frac{\mu}{2}\right)$

38. A launching vehicle carrying an artificial satellite of mass m is set for launch on the surface of the earth of mass M and radius R. If the satellite is intended to move in a circular orbit of radius $7R$, the minimum energy required to be spent by the launching vehicle on the satellite is (Gravitational constant = G)

(a) $\frac{GMm}{R}$ (b) $\frac{13GMm}{14R}$
(c) $\frac{GMm}{7R}$ (d) $\frac{GMm}{14R}$

39. In the circuit shown, the internal resistance of the cell is negligible. The steady state current in the 2 Ω resistor is



(a) 0.6 A (b) 0.9 A
(c) 1.2 A (d) 1.5 A

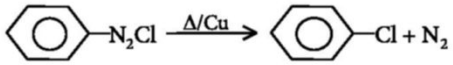
40. Interference fringes were produced in Young's double slit experiment using light of wavelength 5000 Å. When a film of material 2.5×10^{-3} cm thick was placed over one of the slits, the fringe pattern shifted by a distance equal to 20 fringe widths. The refractive index of the material of the film is

(a) 1.25 (b) 1.33 (c) 1.4 (d) 1.5

SECTION-II (CHEMISTRY)

41. The correct decreasing order of acidic character is

(a) HClO > HBrO > HIO
(b) HIO > HBrO > HClO
(c) HBrO > HIO > HClO
(d) HClO > HIO > HBrO

42. 

(A)

Half-life is independent of concentration of A. After 10 minutes volume of N_2 gas is 10 L and after complete reaction 50 L. Hence, rate constant is

(a) $\frac{2.303}{10}\log 5\ \text{min}^{-1}$ (b) $\frac{2.303}{10}\log 1.25\ \text{min}^{-1}$
(c) $\frac{2.303}{10}\log 2\ \text{min}^{-1}$ (d) $\frac{2.303}{10}\log 4\ \text{min}^{-1}$

43. Reduction potentials of some ions are given below. Arrange them in decreasing order of oxidising power.

Ion	ClO_4^-	IO_4^-	BrO_4^-
Reduction potential ($E°$)	1.19 V	1.65 V	1.74 V

(a) $ClO_4^- > IO_4^- > BrO_4^-$ (b) $IO_4^- > BrO_4^- > ClO_4^-$
(c) $BrO_4^- > IO_4^- > ClO_4^-$ (d) $BrO_4^- > ClO_4^- > IO_4^-$

44. The rapid change of pH near the stoichiometric point of an acid-base titration is the basis of indicator detection. pH of the solution is related to the ratio of the concentration of conjugate acid (HIn) and base (In^-) forms of the indicator by the expression

(a) $\log\frac{[In^-]}{[HIn]} = pK_a + pH$

(b) $\log\frac{[HIn]}{[In^-]} = K_a + pH$

(c) $\log\frac{[HIn]}{[In^-]} = pH - pK_a$

(d) $\log\frac{[In^-]}{[HIn]} = pH - pK_a$

45. The coagulation of 100 mL of colloidal solution of gold is completely prevented by addition of 0.25 g of a substance X to it before adding 1 mL of 10% NaCl solution. The gold number of X is

(a) 0.25 (b) 25 (c) 250 (d) 2.5

46. Consider the following statements :

1. Atomic hydrogen is obtained by passing hydrogen through an electric arc.
2. Hydrogen gas will not reduce heated aluminium oxide.
3. Finely divided palladium absorbs large volume of hydrogen gas.
4. Pure nascent hydrogen is best obtained by reacting Na with C_2H_5OH.

Which of the given statements is/are correct?

(a) 1 only (b) 2 only
(c) 1, 2 and 3 (d) 2, 3 and 4

47. The emf of the three galvanic cells given below are represented by E_1, E_2 and E_3.

I. $Zn \mid Zn^{2+}$ (1 M) || Cu^{2+} (1 M) | Cu
II. $Zn \mid Zn^{2+}$ (0.1 M) || Cu^{2+} (1 M) | Cu
III. $Zn \mid Zn^{2+}$ (1 M) || Cu^{2+} (0.1 M) | Cu

Which of the following is true?

(a) $E_1 > E_2 > E_3$ (b) $E_3 > E_2 > E_1$
(c) $E_3 > E_1 > E_2$ (d) $E_2 > E_1 > E_3$

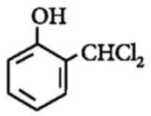
48. Which of the following statements is correct regarding the ions, Zn^{2+}, Ni^{2+} and Cr^{3+} ?
(At. nos. Zn = 30, Ni = 28, Cr = 24)

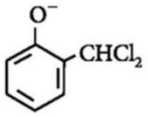
(a) Only Zn^{2+} is colourless, Ni^{2+} and Cr^{3+} are coloured.
(b) All three are colourless.
(c) All three are coloured.
(d) Only Ni^{2+} is coloured, Zn^{2+} and Cr^{3+} are colourless.

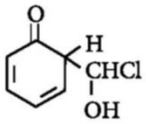
49. An element with molar mass 2.7×10^{-2} kg mol^{-1} forms a cubic unit cell with edge length 405 pm. If its density is 2.7×10^3 kg m^{-3}, what is the nature of the cubic unit cell?

(a) Simple cubic (b) Face-centred
(c) Body-centred (d) End-centred

50. When phenol is reacted with $CHCl_3$ and NaOH followed by acidification, salicylaldehyde is obtained. Which of the following species are involved in the above mentioned reaction as intermediate?

(a) 

(b) 

(c) 

(d) None of these

51. The percentage of hydrogen in H_2O and hydrogen peroxide are respectively 11.2% and 5.94%. This illustrates
(a) law of multiple proportions
(b) conservation of mass
(c) law of definite proportions
(d) law of reciprocal proportions.

52. Which of the following statements is not true?
(a) London smog is a mixture of smoke and fog.
(b) London smog is oxidising in nature.
(c) Photochemical smog causes irritation in eyes.
(d) Photochemical smog results in the formation of PAN.

53. N_2 and O_2 are converted into monocations, N_2^+ and O_2^+ respectively. Which of the following is wrong?
(a) In N_2^+, N – N bond weakens.
(b) In O_2^+, the O – O bond order increases.
(c) In O_2^+, paramagnetism decreases.
(d) N_2^+ becomes diamagnetic.

54. The two isomers *X* and *Y* with the formula $Cr(H_2O)_5ClBr_2$ were taken for experiment on depression in freezing point. It was found that one mole of *X* gave depression corresponding to 2 moles of particles and one mole of *Y* gave depression due to 3 moles of particles. The structural formulae of *X* and *Y* respectively are
(a) $[Cr(H_2O)_5Cl]Br_2$; $[Cr(H_2O)_4Br_2]Cl.\ H_2O$
(b) $[Cr(H_2O)_5Cl]Br_2$; $[Cr(H_2O)_3ClBr_2].\ 2H_2O$
(c) $[Cr(H_2O)_5Br]BrCl$; $[Cr(H_2O)_4ClBr]Br.\ H_2O$
(d) $[Cr(H_2O)_4Br_2]Cl.H_2O$; $[Cr(H_2O)_5Cl]Br_2$

55. Energy of an electron in hydrogen atom is given by $E = -\frac{13.6}{n^2}$ eV. Which one of the following statements is true if *n* is changed from 1 to 3? Energy will
(a) decrease three times (b) increase three times
(c) increase nine times (d) decrease nine times.

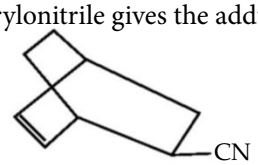
56. Two liquids *A* and *B* have vapour pressures in the ratio $p^\circ_A : p^\circ_B = 1 : 2$ at a certain temperature. Suppose that we have an ideal solutions of *A* and *B* in the mole fractions ratio $A : B = 1 : 2$, the mole fraction of *A* in the vapour in equilibrium with the solution at the given temperature is
(a) 0.25 (b) 0.2 (c) 0.5 (d) 0.33

57. 1500 mL flask contains 400 mg O_2 and 60 mg H_2 at 100°C. What is the total pressure in the flask?
(a) 0.66 atm (b) 0.867 atm
(c) 8.67 atm (d) 13.47 atm

58. In the reaction,
$2X + B_2H_6 \longrightarrow [BH_2(X_2)]^+[BH_4]^-$
'*X*' cannot be
(a) NH_3 (b) CH_3NH_2
(c) $(CH)_3)_2NH$ (d) $(CH_3)_3N$

59. A racemic mixture is a mixture of
(a) *meso* and its isomers
(b) *d*- and its *l*-isomers in equal proportions
(c) *d*- and its *l*- isomers in different proportions
(d) *meso* and *d*-isomers.

60. The Diels—Alder reaction between 1,3-cyclohexadiene and acrylonitrile gives the adduct,



Its IUPAC name is
(a) bicyclo [2.2.2] oct-2-en-5-nitrile
(b) bicyclo [2.2.2] oct-5-en-2-carbonitrile
(c) 3-cyano bicyclo [2.2.2] oct-5-ene
(d) 2-cyano bicyclo [2.2.2] oct-5-ene.

61. Consider the following haloalkanes :
(1) CH_3I (2) CH_3F (3) CH_3Cl (4) CH_3Br
The correct sequence of increasing order of dipole moments is
(a) 1 < 2 < 3 < 4 (b) 4 < 3 < 2 < 1
(c) 1 < 4 < 2 < 3 (d) 3 < 4 < 1 < 2

62. Identify '*Z*' in the given reaction.

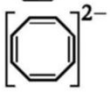
$$C_6H_5NH_2 \xrightarrow{(CH_3CO)_2O} X \xrightarrow{Br_2/CCl_4} Y \xrightarrow[H^+]{HOH} Z$$

(a) *p*-Bromoaniline
(b) *p*-Bromoacetophenone
(c) *p*-Bromoacetanilide
(d) *o*-Bromoacetanilide

63. What will be ΔG for the reaction at 25°C when partial pressures of reactants H_2, CO_2, H_2O and CO are 10, 20, 0.02 and 0.01 atm respectively. (Given : $G^\circ_{H_2O}$ = –228.58 kJ, G°_{CO} = –137.15 kJ, $G^\circ_{CO_2}$ = –394.37 kJ.

(a) +5.61 kJ (b) –5.61 kJ
(c) 7.09 kJ (d) –8.13 kJ

64. Which among the following is aromatic?

(a) (b)
(c) (d) 

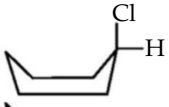
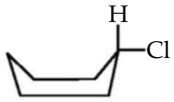
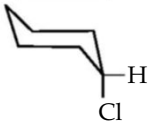
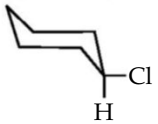
65. In the reaction : $I_2 + 2S_2O_3^{2-} \rightarrow 2I^- + S_4O_6^{2-}$

(a) I_2 is reducing agent.
(b) I_2 is oxidising agent and $S_2O_3^{2-}$ is reducing agent.
(c) $S_2O_3^{2-}$ is oxidising agent.
(d) I_2 is reducing agent and $S_2O_3^{2-}$ is oxidising agent.

66. An ester (*A*) with molecular formula, $C_9H_{10}O_2$ was treated with excess of CH_3MgBr and the complex so formed was treated with H_2SO_4 to give an olefin (*B*). Ozonolysis of (*B*) gave a ketone with molecular formula C_8H_8O which shows positive iodoform test. The structure of (*A*) is

(a) $C_6H_5COOC_2H_5$ (b) $C_6H_5COOC_6H_5$
(c) $CH_3OCH_2COC_6H_5$
(d) *p*-$CH_3OCOC_6H_4COCH_3$

67. The most stable conformation of chlorocyclohexane at room temperature is

(a)  (b) 
(c)  (d) 

68. If *M* is the element of actinide series, the degree of complex formation decreases in the order

(a) $M^{4+} > M^{3+} > MO_2^{2+} > MO_2^+$
(b) $MO_2^+ > MO_2^{2+} > M^{3+} > M^{4+}$
(c) $M^{4+} > MO_2^{2+} > M^{3+} > MO_2{}^+$
(d) $MO_2^{2+} > MO_2{}^+ > M^{4+} > M^{3+}$

69. The value of pK_w of water

(a) increases with increase in temperature
(b) decreases with rise in temperature
(c) does not change with variation in temperature
(d) increases till 50°C and there after decreases.

70. A red coloured oxide (*X*) on treatment with conc. HNO_3 gives a compound (*Y*). (*Y*) with HCl produces a chloride (*Z*) which is insoluble in cold water but soluble in hot water. (*Z*) can also be formed by treating (*X*) with conc. HCl. Compounds *X*, *Y* and *Z* are

(a) Pb_3O_4, PbO_2, $PbCl_2$ (b) Mn_3O_4, MnO_2, $MnCl_2$
(c) Fe_3O_4, Fe_2O_3, $FeCl_3$ (d) Fe_3O_4, FeO, $FeCl_2$

71. The quantum numbers +1/2 and –1/2 for the electron spin represent

(a) rotation of the electron in clockwise and anticlockwise direction respectively
(b) rotation of the electron in anticlockwise and clockwise direction respectively
(c) magnetic moment of the electron pointing up and down respectively
(d) two quantum mechanical spin states which have no classical analogue.

72. 500 mL of a hydrocarbon gas burnt in excess of oxygen yielded 2500 mL of CO_2 and 3.0 L of water vapour. The formula of the hydrocarbon is

(a) C_5H_{12} (b) CH_4 (c) C_2H_4 (d) C_3H_6

73. Which of the following is most reactive towards electrophilic substitution?

(a) Nitrobenzene (b) Aniline
(c) Aniline hydrochloride
(d) *N*-Acetylaniline

74. Antihistamines are not helpful

(a) in curing nasal allergies
(b) in treating rashes caused by itching
(c) in bringing down acute fever
(d) in vasodilation.

75. Match the column I with column II and mark the appropriate choice.

	Column I (Property)	**Column II (Metal)**	
(A)	Element with highest second ionisation enthalpy	(i)	Cr
(B)	Element with highest third ionisation enthalpy	(ii)	Cu
(C)	*M* in $M(CO)_6$ is	(iii)	Zn
(D)	Element with highest heat of atomisation	(iv)	Ni

(a) (A) → (ii), (B) → (iii), (C) → (i), (D) → (iv)
(b) (A) → (iv), (B) → (iii), (C) → (i), (D) → (ii)
(c) (A) → (iii), (B) → (i), (C) → (ii), (D) → (iv)
(d) (A) → (i), (B) → (ii), (C) → (iii), (D) → (iv)

76. Which of the following is not a use of graphite?
(a) For electrodes in batteries.
(b) Crucibles made from graphite are used for its inertness to dilute acids and alkalies.
(c) For adsorbing poisonous gases.
(d) Lubricant at high temperature.

77. *cis*-Polyisoprene possesses elastic property because
(a) it is soft and soluble in non-polar solvent
(b) it is unsaturated and porous
(c) it has a coiled structure and chains held together by weak van der Waals forces
(d) it has a fibrous structure and reactive sites at various double bonds.

78. The number of $sp^3 - s$, $sp^2 - s$, $sp^3 - sp^2$ and $sp^2 - sp^2$ bonds present in 2-butene respectively, are
(a) 2, 1, 1, 4 (b) 6, 2, 2, 1
(c) 2, 3, 4, 6 (d) 2, 2, 6, 1

79. Identify the correct statement for change in Gibbs' energy for a system (ΔG_{system}) at constant temperature and pressure.
(a) If $\Delta G_{system} = 0$, the system has attained equilibrium.
(b) If $\Delta G_{system} = 0$, the system is still moving in a particular direction.
(c) If $\Delta G_{system} = 0$, the process is not spontaneous.
(d) If $\Delta G_{system} = 0$, the process is spontaneous.

80. For which of the following amino acids, van-Slyke estimation method is not applicable?
(a) Alanine (b) Aspartic acid
(c) Serine (d) Proline

SECTION-III (ENGLISH AND LOGICAL REASONING)

Directions (Q. 81–83) : Read the passage and answer the following questions.

In the past thirty years, drugs have been discovered that prevent and cure physical disease and reverse the disturbances that occur in certain mental illness. Excitement over what drugs can do has led people to believe that any ailment, infective or psychic, can be relieved by taking a pill. At the first sign of nervousness, they try pep-up pills. Medical journals now advertise tranquillizers, and other mood-altering drugs; doctors prescribe them; and the public expect miracles from them. In such an atmosphere, it is not surprising that drug abuse has spread.

81. According to the author, in recent years there has been _____ .
(a) recognition of the ill-effects of medicine
(b) a misplaced trust in drugs
(c) a distrust of drugs
(d) none of these

82. According to the passage, the medicines that have been discovered in recent times _____ .
(a) can reduce mental illnesses
(b) cannot cure mental illnesses
(c) can cure mental illnesses
(d) can help treat some symptoms of mental illnesses

83. People often believe that _____ .
(a) medicines can cure all the diseases
(b) doctors cannot cure all the diseases
(c) medicines cannot cure all the diseases
(d) doctors can cure all the diseases

Directions : (Q. 84–86) : Choose the incorrect word in the given sentence.

84. My father is in the bad mood today.
(a) father (b) the (c) mood (d) today

85. The doctor attended to the patient very quietly.
(a) attended (b) the
(c) very (d) no error

86. Supposing if you are arrested, what will you do?
(a) Supposing (b) arrested
(c) will (d) do

Directions (Q. 87–89) : Choose the antonym for the given words from each set.

87. Punctilious
(a) Careless (b) Curious
(c) Irregular (d) Fussy

88. Capacious
(a) Caring (b) Limited
(c) Changeable (d) Foolish

89. Bizarre
(a) Gentle (b) Same (c) Soft (d) Usual

Directions (Q. 90–91) : Choose the synonym for the given words from each set.

90. Diligent
(a) Modest (b) Slack
(c) Industrious (d) Intelligent

91. Vindicate
(a) Convict (b) Fault (c) Judge (d) Justify

Directions (Q. 92–93) : Pick up from answers choice which will complete the sentences correctly.

92. The twins are so alike that I cannot ..... one from the other.
(a) say (b) notice (c) discern (d) tell

93. The chief guest's speech was so ..... that I was all ears.
(a) interesting (b) indeterminate
(c) intricate (d) interminable

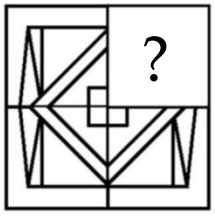
Directions : (Q. 94–95) : Select the pair of words/word, which are related in the same way as the capitalised words are related to each other.

94. INDOLENCE : WORK :: TACITURN : ?
(a) cheat (b) act (c) speak (d) observe

95. SCALES : FISH :: ?
(a) Bear : Fur (b) Woman : Dress
(c) Skin : Man (d) Tree : Leaves

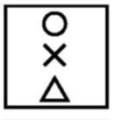
96. A, B, C, D, E, F and G are standing in a queue not necessarily in that order. Each one is wearing a cap of different colour: violet, indigo, blue, green, yellow, orange and red. D is able to see in front of him green and blue but not violet. F can see violet and yellow but not red. G can see caps of all the colours other than orange. If E is wearing indigo then F wears
(a) Blue (b) Red (c) Violet (d) Orange

97. Select a figure from the options which completes the question figure shown below.

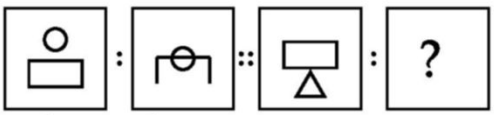


(a)
(b)
(c)
(d)

98. Find the figure that comes in place of '?' and follows/continues the same relationship.

(a) 
(b)
(c)
(d)

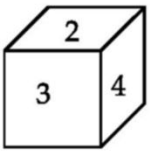
99. If figure on the left of ': :' follows a particular relationship, establish the same relationship on the right as well.

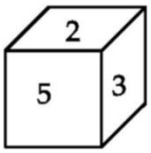


(a)
(b)
(c)
(d)

100. If 23496 = CYSTE and 1578 = BURP, then 85367 = ?
(a) PYSER (b) PUYER
(c) PUYSR (d) PSERY

101. Two positions of a dice is shown below. When 4 is at the bottom, what number will be at the top?





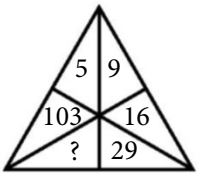
(a) 1 (b) 3 (c) 4 (d) 5

102. Laxman went 15 km to the West from my house, then turned left and walked 20 km. He then turned East and walked 25 km and finally turning left covered 20 km. How far was he from my house?
(a) 5 km (b) 10 km (c) 40 km (d) 80 km

103. Find the mirror image in the given figure.

(a)
(b)
(c)
(d)

104. Find the missing character from the given alternatives.



(a) 54 (b) 55 (c) 56 (d) 57

105. The son of M is the father of N and grandfather (Mother's father) of R. S is the daughter of N and sister of B. On the basis of this information, how is M related to B?
(a) Grandfather (b) Grand mother
(c) Grandmother's mother
(d) Data inadequate

SECTION-IV (MATHEMATICS)

106. The least number of times a fair coin must be tossed so that the probability of getting at least one head is greater than 0.99, is
(a) 7 (b) 8 (c) 9 (d) 6

107. If the line $y = x\sqrt{3}$ cuts the curve $x^3 + y^3 + 3xy + 5x^2 + 3y^2 + 4x + 5y - 1 = 0$ at the points A, B and C, then $OA \cdot OB \cdot OC$ (where O is $(0, 0)$) is
(a) $\frac{4}{13}(3\sqrt{3}-1)$ (b) $3\sqrt{3}+1$
(c) $\frac{1}{\sqrt{3}}(2+7\sqrt{3})$ (d) none of these

108. If $\log_{2^{1/2}} a + \log_{2^{1/4}} a + \log_{2^{1/6}} a + \log_{2^{1/8}} a + \ldots$ upto 20 terms is 840, then a is equal to
(a) 2 (b) 1 (c) 4 (d) $\sqrt{2}$

109. $\lim\limits_{x\to\infty} \frac{\log[x]}{x}$, where $[x]$ is the greatest integer less than or equal to x is
(a) 1 (b) -1
(c) does not exist (d) 0

110. The sum to 212 terms of the series $1 + 4 + 6 + 5 + 11 + 6 + \ldots$ is
(a) 33920 (b) 28355
(c) 33290 (d) none of these

111. The distance between the lines $\vec{r} = (4\hat{i} - 7\hat{j} - 9\hat{k}) + t(3\hat{i} - 7\hat{j} + 4\hat{k})$ and $\vec{r} = (7\hat{i} - 14\hat{j} - 5\hat{k}) + s(-3\hat{i} + 7\hat{j} - 4\hat{k})$ is equal to
(a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 0

112. In a triangle ABC, if $\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$, then $A =$
(a) 30° (b) 45° (c) 60° (d) 90°

113. If α, β, γ are roots of $x^3 + 4x + 1 = 0$, then $(\alpha+\beta)^{-1} + (\beta+\gamma)^{-1} + (\gamma+\alpha)^{-1}$ equals
(a) 2 (b) 3 (c) 4 (d) 5

114. If $f(x) = A\sin\left(\frac{\pi x}{2}\right) + B, f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)dx = \frac{2A}{\pi}$, then the constant A and B are, respectively
(a) $\pi/2$ and $\pi/2$ (b) $2/\pi$ and $3/\pi$
(c) 0 and $-4/\pi$ (d) $4/\pi$ and 0

115. If at any point (x, y) on a curve, subtangent and subnormal are of equal length, then the length of tangent is
(a) $\sqrt{2y}$ (b) $|\sqrt{2}y|$
(c) $|y|$ (d) none of these

116. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, then value of $4x^2 - 4xy\cos\alpha + y^2$ is equal to
(a) 4 (b) $2\sin 2\alpha$
(c) $-4\sin^2\alpha$ (d) $4\sin^2\alpha$

117. If Q is the image of the point $P(2, 3, 4)$ under the reflection in the plane $x - 2y + 5z = 6$, then the equation of the line PQ is
(a) $\frac{x-2}{-1} = \frac{y-3}{2} = \frac{z-4}{5}$ (b) $\frac{x-2}{1} = \frac{y-3}{-2} = \frac{z-4}{5}$
(c) $\frac{x-2}{-1} = \frac{y-3}{-2} = \frac{z-4}{5}$ (d) $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{5}$

118. If the three consecutive coefficients in the expansion of $(1 + x)^n$ are 28, 56 and 70, then 'n' equals
(a) 6 (b) 4 (c) 8 (d) 10

119. Given that $f(x) = x^{1/x}, x > 0$ has the maximum value at $x = e$, then
(a) $e^\pi > \pi^e$ (b) $e^\pi < \pi^e$
(c) $e^\pi = \pi^e$ (d) none of these

120. Range of the function $f(x) = \frac{\text{cosec}^{-1}x + \cos^{-1}(1/x)}{\text{cosec } x}$ is
(a) $(0, \pi)$ (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(c) $\left[-\frac{\pi}{2}, 0\right]$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

121. Twenty passengers are to travel by a double decker bus which can accommodate thirteen in the upper deck and seven in the lower deck. If five of them refuse to travel in the upper deck and eight in the lower deck, then the number of selections that can be made for such a distribution is
(a) 23 (b) 21 (c) 15 (d) 25

122. The equation of the curve whose tangent at any point (x, y) makes an angle $\tan^{-1}(2x + 3y)$ with x-axis and which passes through (1, 2) is
(a) $6x + 9y + 2 = 26e^{3(x-1)}$
(b) $6x - 9y + 2 = 26e^{3(x-1)}$
(c) $6x + 9y - 2 = 26e^{3(x-1)}$
(d) $6x - 9y - 2 = 26e^{3(x-1)}$

123. The mean and standard deviation of 100 observations were calculated as 40 and 5.1, respectively by a student who took by mistake 50 instead of 40 for one observation. Find the correct standard deviation.
(a) 4 (b) 6 (c) 3 (d) 5

124. If $X_1 = (x_1, y_1)$ and $X_2 = (x_2, y_2)$ are two optimal solutions of an L.P.P., then
(a) $\lambda X_1 + (1-\lambda) X_2$, $\lambda \in R$, is also an optimal solution
(b) $\lambda X_1 + (1+\lambda) X_2$, $\lambda \in R$, is also an optimal solution
(c) $\lambda X_1 + (1+\lambda) X_2$, $0 \le \lambda \le 1$, is also an optimal solution
(d) $\lambda X_1 + (1-\lambda) X_2$, $0 \le \lambda \le 1$, is also an optimal solution

125. The function $f(x) = \dfrac{\tan([x-\pi]\pi)}{1+[x]^2}$ is
(a) discontinuous at some x
(b) continuous at all x, but $f'(x)$ does not exist for some x
(c) $f'(x)$ exist for all x, but $f''(x)$ does not exist for some x
(d) $f''(x)$ exists for all x.

126. There are two groups of subjects, group I consists of 5 science subjects and 3 engineering subjects while group II consists of 3 science and 5 engineering subjects. An unbiased die is cast. If number 3 or 5 turns up, a subject from group I is selected, otherwise a subject is selected from group II. The probability that an engineering subject is selected ultimately is
(a) $\dfrac{7}{13}$ (b) $\dfrac{9}{17}$ (c) $\dfrac{13}{24}$ (d) $\dfrac{11}{20}$

127. For any two real numbers a and b, we define aRb if and only if $\sin^2 a + \cos^2 b = 1$. The relation R is
(a) reflexive but not symmetric
(b) symmetric but not transitive
(c) transitive but not reflexive
(d) an equivalence relation

128. If $5\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right) + 7\sin^{-1}\left(\dfrac{2x}{1+x^2}\right) - 4\tan^{-1}\left(\dfrac{2x}{1-x^2}\right) - \tan^{-1} x = 5\pi$, then x is equal to
(a) 3 (b) $\dfrac{1}{3}$ (c) $\dfrac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

129. The locus of the centre of the circle for which one end of diameter is (3, 3) while the other end lies on the line $x + y = 4$ is $x + y = k$, then k equals
(a) 9 (b) 5 (c) 3 (d) 2

130. There are 3 copies each of 4 different books. The number of ways they can be arranged in a shelf is
(a) 369600 (b) 400400 (c) 420600 (d) 440720

131. Let $\vec{v} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{w} = \hat{i} + 3\hat{k}$. If $\vec{u}$ is a unit vector, then the maximum value of the scalar triple product $[\vec{u}\ \vec{v}\ \vec{w}]$ is
(a) -1 (b) $\sqrt{10} + \sqrt{6}$
(c) $\sqrt{59}$ (d) $\sqrt{60}$

132. If $y = (1 + x)(1 + x^2)(1 + x^4) \ldots\ldots (1 + x^{2^n})$ then the value of $\left(\dfrac{dy}{dx}\right)_{x=0}$ is
(a) 0 (b) -1 (c) 1 (d) 2

133. Let $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x. If $a = \sqrt{2011^2 + 2012}$, then the value of $f(a)$ is equal to
(a) 2010 (b) 2011 (c) 2012 (d) 2013

134. For every real number x,
let $f(x) = \dfrac{x}{1!} + \dfrac{3}{2!}x^2 + \dfrac{7}{3!}x^3 + \dfrac{15}{4!}x^4 + \ldots$
Then the equation $f(x) = 0$ has
(a) no real solution
(b) exactly one real solution
(c) exactly two real solutions
(d) infinite number of real solutions

135. $\left(\dfrac{a}{b} - \dfrac{b}{a}\right)\tan\left(i \ln\left(\dfrac{a-ib}{a+ib}\right)\right) =$ ____ where $a, b \in R^+$ and $i = \sqrt{-1}$.
(a) 1 (b) 2 (c) 1/2 (d) -1

136. If $(1 - x + x^2)^n = a_0 + a_1 x + a_2 x^2 + \ldots. + a_{2n} x^{2n}$, then the value of $a_0 + a_2 + a_4 + \ldots\ldots + a_{2n}$ is
(a) $3^n + \dfrac{1}{2}$ (b) $3^n - \dfrac{1}{2}$ (c) $\dfrac{3^n - 1}{2}$ (d) $\dfrac{3^n + 1}{2}$

137. If A is a square matrix of order 3 such that $A^2 + A + 4I = 0$, where 0 is the zero matrix and I is the unit matrix of order 3, then
(a) A is singular and $A + I$ is non-singular
(b) Both A and $A + I$ are non-singular
(c) A is non-singular and $A + I$ is singular
(d) Both A and $A + I$ are singular

138. The entire graph of the equation $y = x^2 + kx - x + 9$ lies above the x-axis, then
(a) $k > 7$ (b) $-5 < k < 7$
(c) $k > -5$ (d) none of these

139. If $a > b > 0$, then maximum value of $\frac{ab(a^2 - b^2)\sin x \cos x}{a^2 \sin^2 x + b^2 \cos^2 x}, x \in \left[0, \frac{\pi}{2}\right]$ is
(a) $a^2 - b^2$ (b) $\frac{a^2 - b^2}{2}$
(c) $\frac{a^2 + b^2}{2}$ (d) none of these

140. Let $\vec{a} = \alpha_1 \hat{i} + \alpha_2 \hat{j} + \alpha_3 \hat{k}$, $\vec{b} = \beta_1 \hat{i} + \beta_2 \hat{j} + \beta_3 \hat{k}$ and $\vec{c} = \gamma_1 \hat{i} + \gamma_2 \hat{j} + \gamma_3 \hat{k}$, $|\vec{a}| = 2\sqrt{2}$. If $\vec{a}$ makes an angle $\frac{\pi}{3}$ with plane of $\vec{b}$ and $\vec{c}$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix}^n$ is equal to (n is even natural number)
(a) $\left(\frac{|\vec{b}||\vec{a}|}{2 \times 3}\right)^{n/2}$ (b) $\left(\frac{\sqrt{3}|\vec{b}||\vec{c}|}{\sqrt{2}}\right)^n$
(c) $\frac{(|\vec{b}||\vec{c}|)^{n/2}}{\sqrt{3}\,2^n}$ (d) none of these

141. If $\int \frac{2e^x + 3e^{-x}}{3e^x + 4e^{-x}} dx = Ax + B \log(3e^{2x} + 4)$, then
(a) $A = -\frac{3}{4}, B = \frac{1}{24}$ (b) $A = \frac{3}{4}, B = -\frac{1}{24}$
(c) $A = \frac{1}{4}, B = \frac{1}{24}$ (d) $A = -\frac{3}{4}, B = \frac{1}{4}$

142. The tangent at any point on the curve $x^4 + y^4 = a^4$ cuts off intercepts p and q on the co-ordinate axes then the value of $p^{-4/3} + q^{-4/3}$ is equal to
(a) $a^{-4/3}$ (b) $a^{-1/2}$
(c) $a^{1/2}$ (d) none of these

143. If 1, ω, ω^2 are cube roots of unity and if $= \begin{bmatrix} 1+\omega & 2\omega \\ -2\omega & -b \end{bmatrix} + \begin{bmatrix} a & -\omega \\ 3\omega & 2 \end{bmatrix} = \begin{bmatrix} 0 & \omega \\ \omega & 1 \end{bmatrix}$, then $a^2 + b^2$
(a) $1 + \omega^2$ (b) $\omega^2 - 1$ (c) $1 + \omega$ (d) ω^2

144. The area bounded by the curves $y = f(x)$, x-axis and the ordinates $x = 1$ and $x = b$ is $(b - 1)\sin(3b + 4)$. Then $f(x)$ is
(a) $(x - 1)\cos(3x + 4)$ (b) $\sin(3x + 4)$
(c) $\sin(3x + 4) + 3(x - 1)\cos(3x + 4)$
(d) none of these

145. Let $P(6, 3)$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x-axis at (9, 0) then the eccentricity of the hyperbola is
(a) $\sqrt{\frac{5}{2}}$ (b) $\sqrt{\frac{3}{2}}$ (c) $\sqrt{2}$ (d) $\sqrt{3}$

146. Which of the following statements is false?
(a) The feasible region is always a concave region.
(b) The maximum (or minimum) solution of the objective function occurs at the vertex of the feasible region.
(c) If two corner points produce the same maximum (or minimum) value of the objective function, then every point on the line segment joining these points will also give the same maximum (or minimum) value.
(d) All of the above

147. If $\frac{dy}{dx} = \frac{x - y}{x + y}$, $y(1) = 1$, then $(y(0))^2 =$
(a) 1 (b) 2 (c) 3 (d) 4

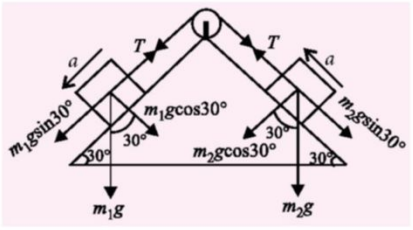
148. In a ΔABC, a, b, c are the sides of the triangle opposite to the angles A, B, C respectively. Then the value of $a^3 \sin(B - C) + b^3 \sin(C - A) + c^3 \sin(A - B)$ is equal to
(a) 0 (b) 1 (c) 3 (d) 2

149. Let O be the origin and A be the point (64, 0). If P, Q divide OA in the ratio 1 : 2 : 3, then the point P is
(a) $\left(\frac{32}{3}, 0\right)$ (b) (32, 0)
(c) $\left(\frac{64}{3}, 0\right)$ (d) (16, 0)

150. The function $f: R^+ \to (1, e)$ defined by $f(x) = \frac{x^2 + e}{x^2 + 1}$ is
(a) one-one but not onto
(b) onto but not one-one
(c) both one-one and onto
(d) neither one-one nor onto

SOLUTIONS

1. (d) :



$m_1g\sin30° - T = m_1a$ ... (i)

$T - m_2g\sin30° = m_2a$ ... (ii)

Adding (i) and (ii), $a = \dfrac{m_1g\sin30° - m_2g\sin30°}{m_1 + m_2}$

From eqn. (ii), we get

$T = m_2g\sin30° + m_2a$

$= m_2g\sin30° + \dfrac{m_2}{m_1+m_2}(m_1g\sin30° - m_2g\sin30°)$

$= \dfrac{2m_1m_2g\sin30°}{m_1+m_2} = \dfrac{2\times40\times30\times9.8\times(1/2)}{40+30}$

$= \dfrac{1200}{70}\times9.8 = 168$ N

2. (b) : Here, $T_{1/2} = 5.25$ years, $t = 21$ years

No. of half lives, $n = \dfrac{t}{T_{1/2}} = \dfrac{21 \text{ years}}{5.25 \text{ years}} = 4$

Amount of sample left

$= \left(\dfrac{1}{2}\right)^4 \times 1 \text{ g} = \dfrac{1}{16} \text{ g} = \dfrac{1000}{16} \text{ mg} = 62.5 \text{ mg}$

3. (b) : At constant temperature, pV = constant

$p_1V_1 = p_2V_2$ or $\dfrac{p_1}{p_2} = \dfrac{V_2}{V_1}$

Fractional change in volume

$\dfrac{V_1 - V_2}{V_1} = \dfrac{4}{100} = \dfrac{1}{25} \Rightarrow \dfrac{V_2}{V_1} = \dfrac{24}{25}$

$\therefore \dfrac{p_1}{p_2} = \dfrac{V_2}{V_1} = \dfrac{24}{25}$ or $\dfrac{p_2}{p_1} = \dfrac{25}{24} \Rightarrow \dfrac{p_2 - p_1}{p_1} = \dfrac{1}{24}$

% increase in pressure = 100/24 = 4.17 %

4. (a) : The magnetic field at a point on the axis of circular current carrying loop is given by $B = \dfrac{\mu_0}{4\pi}\dfrac{2\pi IR^2}{(R^2+x^2)^{3/2}}$

where R is the radius of the loop, x is the distance of the point on the axis from the centre of the loop

Here, $R = 3$ cm $= 3 \times 10^{-2}$ m, $x = 4$ cm $= 4 \times 10^{-2}$ m, $B = 54 \times 10^{-6}$ T

$\therefore \quad 54\times10^{-6} = \dfrac{\mu_0}{4\pi}\dfrac{2\pi I(3\times10^{-2})^2}{(3^2+4^2)^{3/2}\times10^{-6}} = \dfrac{\mu_0}{4\pi}\dfrac{2\pi I9}{125\times10^{-2}}$ ...(i)

Magnetic field at the centre of the loop is

$B' = \dfrac{\mu_0}{4\pi}\dfrac{2\pi I}{R} = \dfrac{\mu_0}{4\pi}\dfrac{2\pi I}{3\times10^{-2}}$ ...(ii)

Dividing (ii) by (i), we get

$\dfrac{B'}{54\times10^{-6}} = \dfrac{125\times10^{-2}}{9\times3\times10^{-2}}$ or $B' = 250 \times 10^{-6}$ T $= 250$ μT

5. (a) : Here, Input resistance, $R_i = 100$ Ω

Change in base current, $\Delta I_B = 40$ μA

Change in collector current, $\Delta I_C = 2$ mA

Load resistance, $R_L = 4$ kΩ $= 4 \times 10^3$ Ω

Current gain, $\beta = \dfrac{\Delta I_C}{\Delta I_B} = \dfrac{2 \text{ mA}}{40 \text{ μA}} = \dfrac{2\times10^{-3} \text{ A}}{40\times10^{-6} \text{ A}} = 50$

Voltage gain of the amplifier is

$A_V = \beta\dfrac{R_L}{R_i} = 50\times\dfrac{4\times10^3}{100} = 2000$

6. (a) : Compare $e = 220\sqrt{2}\sin100t$ V with $e = e_0\sin\omega t$, we get $e_0 = 220\sqrt{2}$ V, $\omega = 100$ rad s^{-1}

$e_{rms} = \dfrac{e_0}{\sqrt{2}} = \dfrac{220\sqrt{2}}{\sqrt{2}}$ V $= 220$ V

The capacitive reactance is

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{100\times1\times10^{-6}} = 10^4$ Ω

The current flowing through the capacitor is

$I_{rms} = \dfrac{e_{rms}}{X_C} = \dfrac{220V}{10^4\,\Omega} = 220\times10^{-4}\text{A} = 22$ mA

7. (c)

8. (c) : $x = A\sin(\omega t + \pi/2) = A\cos\omega t$

$\therefore \quad \cos\omega t = x/A$ and $\sin\omega t = \sqrt{1-(x^2/A^2)}$

$y = A\sin2\omega t = 2A\sin\omega t\cos\omega t$

or $y^2 = 4A^2\sin^2\omega t\cos^2\omega t$

$= 4A^2\times\dfrac{x^2}{A^2}\times\left(\dfrac{A^2-x^2}{A^2}\right) = 4x^2\left(1-\dfrac{x^2}{A^2}\right)$

9. (d) : 1 calorie = 4.2 J

$\therefore$ [calorie] = $[ML^2T^{-2}]$.

Comparing with general dimensional formula $[M^aL^bT^c]$, we get $a = 1$, $b = 2$, $c = -2$

$\because \quad n_2 = n_1\left[\dfrac{M_1}{M_2}\right]^a\left[\dfrac{L_1}{L_2}\right]^b\left[\dfrac{T_1}{T_2}\right]^c$

$\Rightarrow \quad n_2 = 4.2\left[\dfrac{1 \text{ kg}}{\alpha \text{ kg}}\right]^1\left[\dfrac{1 \text{ m}}{\beta \text{ m}}\right]^2\left[\dfrac{1 \text{ s}}{\gamma \text{ s}}\right]^{-2} = 4.2\,\alpha^{-1}\beta^{-2}\gamma^2$

10. (a) : The hydrogen atom in the ground state (–13.6 eV) must be excited to the energy level with $n = 4$ (*i.e.*, – 0.85 eV) so as to have H_β line.

11. (b)

12. (a) : Applying principle of conservation of linear momentum,

$\dfrac{m}{20}\times v = \left(m + \dfrac{m}{20}\right)v' = \dfrac{21m}{20}v' \Rightarrow v' = \dfrac{v}{20}\times\dfrac{20}{21} = \dfrac{v}{21}$

This is the velocity gained by the bag.

Initial kinetic energy of bullet, $E_1 = \dfrac{1}{2}\left(\dfrac{m}{20}\right)v^2$

Final kinetic energy of bag and bullet,

$E_2 = \dfrac{1}{2}\left(m + \dfrac{m}{20}\right)v'^2 = \dfrac{1}{2}\left(\dfrac{21m}{20}\right)\left(\dfrac{v}{21}\right)^2$

Loss of energy

$$= E_1 - E_2 = \frac{1}{2}\left(\frac{m}{20}\right)v^2\left[1-\frac{1}{21}\right] = \frac{1}{2}\left(\frac{m}{20}\right)v^2 \times \frac{20}{21}$$

Fraction of energy lost $= \frac{E_1 - E_2}{E_1} = \frac{20}{21}$

13. (b) : $\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{\Delta I_C}{\Delta I_E - \Delta I_C}$ $(\because\ \Delta I_E = \Delta I_B + \Delta I_C)$

or $100 = \frac{1}{\Delta I_E - 1}$ or $\Delta I_E - 1 = \frac{1}{100} = 0.01$ mA

or $\Delta I_E = (1 + 0.01)$ mA $= 1.01$ mA

14. (b)

15. (c) : Work function, $\phi_0 = \frac{hc}{\lambda_{longest}}$

or $\lambda_{longest} = \frac{hc}{\phi_0} = \frac{(6.63\times10^{-34})\times(3\times10^8)}{4.0\times1.6\times10^{-19}}$

$\approx 310 \times 10^{-9}$ m $= 310$ nm

16. (b) : The displacement current is given by

$$I_d = \varepsilon_0 \frac{d\phi}{dt} = \varepsilon_0 \frac{d}{dt}(AE) = \frac{\varepsilon_0 A}{d}\frac{dV}{dt} \qquad \left(\because\ E = \frac{V}{d}\right)$$

$$I_d = C\frac{dV}{dt} \qquad \left(\because\ C = \frac{\varepsilon_0 A}{d}\right)$$

Substituting the given values, we get

$I_d = 2 \times 10^{-6} \times 10^6 = 2$ A

17. (c) : For sonometer, frequency of vibration of string, provided its tension and mass per unit length remain same, is given by $\upsilon \propto \frac{1}{l}$

$\therefore\ \frac{\upsilon_2}{\upsilon_1} = \frac{l_1}{l_2}$ or $\frac{\upsilon_2}{256} = \frac{25}{16}$ $\Rightarrow$ $\upsilon_2 = \frac{256\times25}{16} = 400$ Hz

18. (a) **19. (c)**

20. (b) : Here, $B = B_0 e^{-\frac{t}{\tau}}$

Area of the circular loop, $A = \pi r^2$

Flux linked with the loop at any time, t,

$$\phi = BA = \pi r^2 B_0 e^{-\frac{t}{\tau}}$$

Emf induced in the loop, $\varepsilon = -\frac{d\phi}{dt} = \pi r^2 B_0 \frac{1}{\tau} e^{-t/\tau}$

Net heat generated in the loop

$$= \int_0^\infty \frac{\varepsilon^2}{R}dt = \frac{\pi^2 r^4 B_0^2}{\tau^2 R}\int_0^\infty e^{-\frac{2t}{\tau}}dt = \frac{\pi^2 r^4 B_0^2}{\tau^2 R}\times\frac{1}{\left(-\frac{2}{\tau}\right)}\times\left[e^{-\frac{2t}{\tau}}\right]_0^\infty$$

$$= \frac{-\pi^2 r^4 B_0^2}{2\tau^2 R}\times\tau(0-1) = \frac{\pi^2 r^4 B_0^2}{2\tau R}$$

21. (b) : According to equation of continuity,

$a_1 v_1 = a_2 v_2$

$\therefore\ (40\times10^{-8})\times v_1 = 8\times10^{-4}\times\left(\frac{0.15}{60}\right)$

or $v_1 = \frac{8\times10^{-4}\times0.15}{40\times10^{-8}\times60} = 5$ m s^{-1}

22. (b) : Let h be maximum height reached by the body. Taking motion of the body from half the maximum height upto the highest point, we have

$u = 10$ m s^{-1}, $a = -g = -10$ m s^{-2}, $v = 0$, $S = h/2$

As $v^2 = u^2 + 2aS$

$\therefore\ 0 = 10^2 + 2(-10)\times h/2$ or $h = 10$ m

23. (c) : According to Kepler's third law $T^2 \propto R^3$

$$T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2} = 4\left(\frac{3R}{R}\right)^{3/2} = 4\sqrt{27}\ \text{hours}$$

24. (c) : For SHM, Maximum velocity, $v_{max} = A\omega$

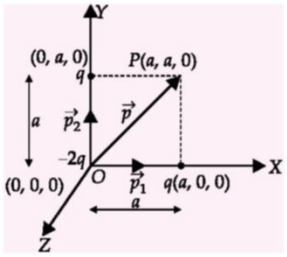
For 1st case, $v_{max_1} = A_1\omega_1 = A_1\sqrt{\frac{k_1}{M}}$ ...(i)

For IInd case, $v_{max_2} = A_2\omega_2 = A_2\sqrt{\frac{k_2}{M}}$ ...(ii)

According to question, $v_{max_1} = v_{max_2}$

$\therefore\ A_1\sqrt{\frac{k_1}{M}} = A_2\sqrt{\frac{k_2}{M}}$ or $\frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$

25. (a) : The given charge assembly is equivalent to two dipoles. One dipole of charges $-q$ and $+q$ has dipole moment $\vec{p}_1$ along positive X-direction and other dipole of charges $-q$ and $+q$ has dipole moment $\vec{p}_2$ along positive Y-direction.



The resultant dipole moment $\vec{p}$ has magnitude,

$p = \sqrt{p_1^2 + p_2^2} = \sqrt{q^2a^2 + q^2a^2} = \sqrt{2}qa$

$\vec{p}$ is directed along $\overrightarrow{OP}$, where P is $(a, a, 0)$.

26. (c)

27. (c) : Potential difference across wire $= I\times3 = \frac{2\times3}{(3+R)}$

Potential gradient $= \frac{6}{(3+R)}\frac{1}{100} = 10^{-3}$ V cm^{-1} (Given)

or $60 = 3 + R$ or $R = 57\ \Omega$

28. (b) : Here, $d = 1$ mm $= 10^{-3}$ m, $\lambda = 6.5\times10^{-7}$ m, $D = 1$ m

$$x_5 = n\lambda\frac{D}{d} = 5\times6.5\times10^{-7}\times\frac{1}{10^{-3}} = 32.5\times10^{-4}\ \text{m}$$

$$x_3 = (2n-1)\frac{\lambda}{2}\frac{D}{d} = \frac{(2\times3-1)\times6.5\times10^{-7}\times1}{2\times10^{-3}} = 16.25\times10^{-4}\ \text{m}$$

$x_5 - x_3 = (32.5 - 16.25)\times10^{-4}$ m $= 16.25\times10^{-4}$ m $= 1.63$ mm

29. (c): Due to change in temperature t°C, increase in length,

$\Delta l = l\alpha t$ or $\frac{\Delta l}{l} = \alpha t$

$Y = \frac{\text{stress}}{\text{strain}} = \frac{\text{stress}}{\Delta l/l}$ $\therefore$ Stress $= Y\times\frac{\Delta l}{l} = Y\alpha t$

30. (d) : Resistance of coil, $R = \frac{200\ \text{V}}{1\ \text{A}} = 200\ \Omega$

With ac source, $I = \frac{200}{\sqrt{R^2 + X_L^2}}$ or $0.5 = \frac{200}{\sqrt{R^2 + X_L^2}}$

or $R^2 + (2\pi\upsilon L)^2 = (400)^2$

or $\left(2\pi\upsilon \times \frac{2\sqrt{3}}{\pi}\right)^2 = (400)^2 - (200)^2 = 200 \times 600$

or $4\sqrt{3}\upsilon = 2\sqrt{3} \times 100$ or $\upsilon = 50$ Hz.

31. (a): Here, $u_o = -5$ cm, $f_o = 4$ cm, $f_e = 10$ cm, $D = 20$ cm

According to lens formula

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o} \quad \text{or} \quad \frac{1}{v_o} = \frac{1}{f_o} + \frac{1}{u_o}$$

Substituting the given values, we get

$$\frac{1}{v_o} = \frac{1}{4} + \frac{1}{-5} = \frac{1}{4} - \frac{1}{5} = \frac{1}{20}$$

$v_o = 20$ cm

Magnification, $M = \frac{v_o}{|u_o|}\left(1 + \frac{D}{f_e}\right) = \frac{20}{5}\left(1 + \frac{20}{10}\right) = 12$ cm

32. (b) : $\frac{3h}{2\pi} = n\left(\frac{h}{2\pi}\right) \quad \therefore \quad n = 3$

The kinetic energy of the electron in n^{th} orbit is

$K_n = \frac{13.6}{n^2}$ eV $\quad \therefore \quad K_3 = \frac{13.6}{3^2}$ eV $= \frac{13.6}{9}$ eV $= 1.51$ eV

33. (a) : Initial fundamental frequency of a stretched string is

$$\upsilon = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \qquad \ldots(i)$$

where the symbols have their usual meanings.

When the length of a stretched string is shortened by 40% and the tension is increased by 44%, then its length and tension become

$$L' = L - \frac{40}{100}L = \frac{3}{5}L, \quad T' = T + \frac{44}{100}T = \frac{36}{25}T$$

Then, final fundamental frequency is

$$\upsilon' = \frac{1}{2L'}\sqrt{\frac{T'}{\mu}} = \frac{1}{2\left(\frac{3}{5}L\right)}\sqrt{\frac{36}{25}\frac{T}{\mu}} = \frac{2}{2L}\sqrt{\frac{T}{\mu}} \qquad \ldots(ii)$$

Dividing (ii) by (i), we get $\frac{\upsilon'}{\upsilon} = \frac{2}{1}$

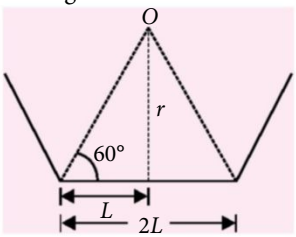
34. (a) : Length of each side of hexagon $= 2L$

Mass of each side $= m$

Let O be centre of mass of hexagon.

Therefore, perpendicular distance of O from each side

$r = L\tan 60° = L\sqrt{3}$



The desired moment of inertia of hexagon about O is

$I = 6\,[I_{\text{one side}}]$

$$= 6\left[\frac{m(2L)^2}{12} + mr^2\right] = 6\left[\frac{mL^2}{3} + 3mL^2\right] = 20\,mL^2$$

35. (c) : Mutual inductance of the system,

$M = \mu_0 n_1 n_2 A_2 l$

where A_2 is the area of inner solenoid.

$n_1 = 10\ cm^{-1} = 1000\ m^{-1}$, $n_2 = 40\ cm^{-1} = 4000\ m^{-1}$

$l = 30$ cm $= 30 \times 10^{-2}$ m, $A_2 = 20\ cm^2 = 20 \times 10^{-4}\ m^2$

$\therefore \quad M = 4\pi \times 10^{-7} \times 1000 \times 4000 \times 20 \times 10^{-4} \times 30 \times 10^{-2}$

$= 301.44 \times 10^{-5}$ H $= 3$ mH

36. (c)

37. (c) : As $\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

Here, $\delta_m = A$

$$\therefore \quad \mu = \frac{\sin A}{\sin\left(\frac{A}{2}\right)} = \frac{2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = 2\cos\left(\frac{A}{2}\right)$$

$$A = 2\cos^{-1}\left(\frac{\mu}{2}\right)$$

38. (b) : The energy of the satellite on the surface of the earth is

$$E_s = \text{KE} + \text{PE} = 0 + \left(-\frac{GMm}{R}\right) = -\frac{GMm}{R}$$

The energy of the satellite in an orbit of radius r is

$$E_o = \frac{1}{2}mv_o^2 + \left(-\frac{GMm}{r}\right)$$

$$= \frac{1}{2}m\left(\frac{GM}{r}\right) - \frac{GMm}{r} = -\frac{GMm}{2r}$$

The minimum energy required to be spent by the vehicle is

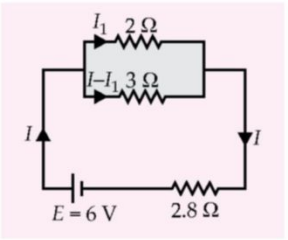
$\Delta E = E_o - E_s$

$$= -\frac{GMm}{2(7R)} - \left[-\frac{GMm}{R}\right] = -\frac{GMm}{14R} + \frac{GMm}{R} = \frac{13}{14}\frac{GMm}{R}$$

39. (b) : Capacitance of the capacitor (C) = 0.2 μF and emf of cell (E) = 6 V.

Reactance of a capacitor for a cell, which is a DC source, is infinity. Therefore, no current flows in 4 Ω resistance.

Resistances 2 Ω and 3 Ω (both in upper arm) are connected in parallel.



Therefore, their equivalent resistance $(R') = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$

Now, R' and 2.8 Ω are in series combination. Therefore, equivalent resistance of the circuit,

$R = R' + 2.8 = 1.2 + 2.8 = 4\ \Omega$

Current drawn in the circuit, $I = \frac{E}{R} = \frac{6}{4} = 1.5$ A

From figure,

$I_1 \times 2 = (I - I_1) \times 3 \quad \Rightarrow \quad 2I_1 = 3I - 3I_1 \quad \Rightarrow \quad 5I_1 = 3I$

$$\Rightarrow \quad I_1 = \frac{3 \times 1.5}{5} = 0.9 \text{ A}$$

40. (c) : Fringe width $\beta = \frac{\lambda D}{d}$ ...(i)

where D is the distance between the screen and slit and d is the distance between two slits.

When a film of thickness t and refractive index μ is placed over one of the slit, the fringe pattern is shifted by distance S and is given by

$$S = \frac{(\mu - 1)tD}{d} \quad \ldots(ii)$$

Given, $S = 20\beta$ ...(iii)

From equations (i), (ii) and (iii) we get,

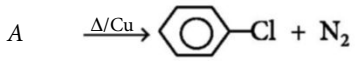
$(\mu - 1)t = 20\lambda$

or $(\mu - 1) = \frac{20\lambda}{t} = \frac{20 \times 5000 \times 10^{-8}\ \text{cm}}{2.5 \times 10^{-3}\ \text{cm}}$

$\mu - 1 = 0.4$ or $\mu = 1.4$

41. (a) : Acidity decreases as the electronegativity of the central halogen decreases from Cl to I in HXO (oxoacids).

42. (b) : It follows first order kinetics since half-life is independent of concentration.



$t = 0$	a	0	0
$t = 10$ min	$(a - x)$	x	x
complete	$(a - a)$	a	a

Hence, $x = 10$ L, $a = 50$ L $\therefore k = \frac{2.303}{10}\log\frac{50}{40}\ \text{min}^{-1}$

$k = \frac{2.303}{10}\log 1.25\,\text{min}^{-1}$

43. (c) : Higher the reduction potential value of a species, greater is its tendency to undergo reduction and stronger is the oxidising power.

	BrO_4^-	IO_4^-	ClO_4^-
$E°$/V	1.74	1.65	1.19

44. (d) : For $HIn \rightleftharpoons H^+ + In^-$

$$K_a = \frac{[H^+][In^-]}{[HIn]}$$

$\therefore \quad pH = pK_a + \log\frac{[In^-]}{[HIn]}$ or $pH - pK_a = \log\frac{[In^-]}{[HIn]}$

45. (b) : 250 mg of X is present in 100 mL of colloidal sol of gold. By definition, gold number of X is that amount of it in mg which is present in 10 mL of colloidal gold solution. Hence, in 10 mL the amount of X present is 25 mg, which is the gold number of X.

46. (c) : Pure hydrogen is evolved by reacting absolute alcohol and Na.

$C_2H_5OH + Na \rightarrow C_2H_5ONa + 1/2H_2$

Other statements are correct.

47. (d) : $E_{cell} = E^\circ_{cell} - \frac{RT}{nF}\ln\frac{[Zn^{2+}]}{[Cu^{2+}]}$

For I, $\frac{[Zn^{2+}]}{[Cu^{2+}]} = 1$, $\therefore \quad E_{cell} = E^\circ_{cell}$

For II, $\frac{[Zn^{2+}]}{[Cu^{2+}]} = \frac{0.1}{1}$

$\therefore \quad E_{cell} = E^\circ_{cell} - \frac{0.0591}{n}\log 10^{-1} = E^\circ_{cell} + \frac{0.0591}{n}$

For III, $\frac{[Zn^{2+}]}{[Cu^{2+}]} = \frac{1}{0.1} = 10$

$\therefore \quad E_{cell} = E^\circ_{cell} - \frac{0.0591}{n}\log 10 = E^\circ_{cell} - \frac{0.0591}{n}$

$\therefore \quad E_2 > E_1 > E_3$

48. (a) : Electronic configuration of $Zn^{2+} = [Ar]3d^{10}4s^0$; $Ni^{2+} = [Ar]3d^8 4s^0$; $Cr^{3+} = [Ar]\ 3d^3\ 4s^0$. Hence, Zn^{2+} is colourless (no unpaired electrons), Ni^{2+} and Cr^{3+} are coloured due to unpaired electrons.

49. (b) : Number of atoms present in the unit cell

$$Z = \frac{d \times a^3 \times N_A}{M}$$

$$Z = \frac{(2.7 \times 10^3\ \text{kg m}^{-3})(4.05 \times 10^{-10}\ \text{m})^3 \times (6.022 \times 10^{23}\ \text{mol}^{-1})}{(2.7 \times 10^{-2}\ \text{kg mol}^{-1})} = 4$$

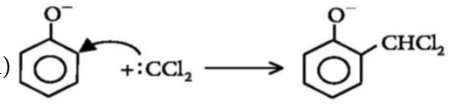
Since, there are four atoms per unit cell, the cubic unit cell must be face-centred.

50. (b) : The given reaction is known as Reimer-Tiemann reaction. The formation of intermediates can be explained through its mechanism.

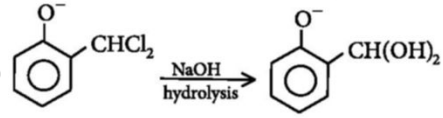
(i) $OH^- + CHCl_3 \rightleftharpoons HOH + :CCl_3^- \longrightarrow Cl^- + :\underset{\text{Dichloro-carbene}}{CCl_2}$

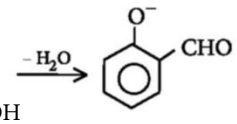
(ii) $C_6H_5OH \xrightarrow[-H^+]{OH^-} C_6H_5O^-$

(iii)

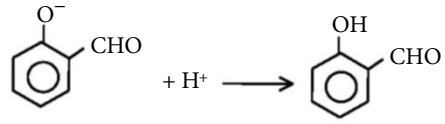


(iv)





(v)



Salicylaldehyde

51. (a) **52. (b)**

53. (d) : (O_2 = 16 electrons)

$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2, \pi 2p_y^2, \pi^* 2p_x^1, \pi^* 2p_y^1$

(N_2^+ =13 electrons)

$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi 2p_x{}^2, \pi 2p_y{}^2, \sigma 2p_z{}^1$

Thus, it contains one unpaired electron, hence paramagnetic.

54. (d)

55. (d) : $E \propto \frac{1}{n^2}$ *i.e.*, when $n = 3$; E decreases nine times.

56. (b) : Since the ratio of p_A° and p_B° is 1 : 2 and mole fraction is 1 : 2 therefore, partial pressure of A (p_A')

$= p_A^\circ x_A$ and partial pressure of B $(p_B') = p_B^\circ\ x_B$ are related as

$p'_B = 4p'_A$

$P = p'_A + p'_B = p'_A + 4p'_A = 5p'_A$

The mole fraction of A in the vapour in equilibrium with solution (according to Dalton's law of partial pressure) is

$$x'_A = \frac{p'_A}{P} = \frac{p'_A}{5p'_A} = \frac{1}{5} = 0.2$$

57. (b) : Number of moles of $O_2 = \frac{w}{M} = \frac{400\times10^{-3}}{32} = 0.0125$

Number of moles of $H_2 = \frac{w}{M} = \frac{60\times10^{-3}}{2} = 0.03$

From, $pV = nRT$

Partial pressure of $O_2 = \frac{0.0125\times0.0821\times373}{1500\times10^{-3}} = 0.255$ atm

Partial pressure of $H_2 = \frac{0.03\times0.0821\times373}{1500\times10^{-3}} = 0.612$ atm

Total pressure = 0.255 + 0.612 = 0.867 atm

58. (d) : $B_2H_6 + 2N(CH_3)_3 \longrightarrow 2BH_3{\cdot}N(CH_3)_3$

59. (b) : A mixture of 50% of *d* and 50% of *l* form is called racemic mixture.

60. (b)

61. (c) : $\underset{1.6\ D}{CH_3I} < \underset{1.79\ D}{CH_3Br} < \underset{1.84\ D}{CH_3F} < \underset{1.94\ D}{CH_3Cl}$

The charge separation in C – *X* bond decreases in the order : C – F > C – Cl > C – Br > C – I

due to decrease in the electronegativity as we move from F → Cl → Br → I. CH_3Cl has higher dipole moment than CH_3F because of larger bond length of C – Cl bond. The larger C – Cl bond length causes dipole moment (μ) = $q \times d$ to be larger for CH_3Cl than CH_3F. In case of other haloalkanes, the effect of larger C – *X* bond length cannot out weight the effect of decreased charge separation.

62. (a)

63. (b) : $H_{2(g)} + CO_{2(g)} \rightleftharpoons H_2O_{(g)} + CO_{(g)}$

ΔG° for reaction = $[G^\circ_{H_2O} + G^\circ_{CO}] - [G^\circ_{H_2} + G^\circ_{CO_2}]$

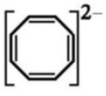
= [–228.58 – 137.15] – [0 – 394.37] = 28.64 kJ

Also, $\Delta G = \Delta G^\circ + 2.303\ RT \log Q$

$= 28.64 + 2.303 \times 8.314 \times 10^{-3} \times 298 \log \frac{0.02\times0.01}{10\times20}$

ΔG = –5.61 kJ

Thus, reaction will occur in forward direction.

64. (d) : Compound  has 8 + 2 = 10π-electrons, hence it is aromatic.

has $4\pi e^-$ and has $8\pi e^-$ then, these are antiaromatic compounds while has 8 + 1 = $9\pi e^-$, hence, it is non-aromatic.

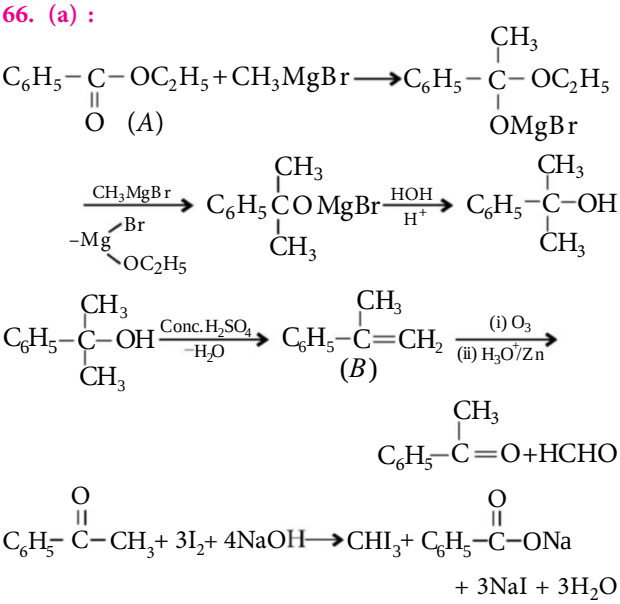
65. (b) : $\overset{0}{I_2} \rightarrow \overset{-1}{2I^-}$

I_2 undergoes reduction hence acts as oxidising agent.

$\overset{+2}{S_2}O_3^{2-} \rightarrow \overset{+2.5}{S_4}O_6^{2-}$

It undergoes oxidation hence acts as a reducing agent.

66. (a) :



67. (d) : In this conformer Cl is at equatorial position and is least hindered.

68. (c): The higher the charge on the metal ion, smaller is the ionic size and more is the complex forming ability. Thus, the degree of complex formation decreases in the order :

$$M^{4+} > MO_2^{2+} > M^{3+} > MO_2^+$$

The higher tendency of complex formation of MO_2^{2+} as compared to M^{3+} is due to high concentration of charge on metal atom M in MO_2^{2+}.

69. (b) : On increasing temperature, K_w of water increases and hence pK_w decreases.

70. (a) : Compound *X*, *Y* and *Z* are Pb_3O_4, PbO_2 and $PbCl_2$.

$\underset{(X)}{Pb_3O_4} + 4HNO_3 \rightarrow 2Pb(NO_3)_2 + \underset{(Y)}{PbO_2} + 2H_2O$

$PbO_2 + 4HCl \rightarrow \underset{(Z)}{PbCl_2} + 2H_2O + Cl_2$

$PbCl_2$ is insoluble in cold water but soluble in hot water.

$\underset{(X)}{Pb_3O_4} + 8HCl \rightarrow \underset{(Z)}{3PbCl_2} + 4H_2O + Cl_2$

71. (d) : Magnetic moment for an electron exists even if the orbital angular momentum is zero. This is explained by the spin magnetic moment because it is the spin which produces magnetic moment.

72. (a) : $C_xH_y + \left[x + \frac{y}{4}\right]O_2 \rightarrow x\,CO_2 + \frac{y}{2}H_2O_{(v)}$

Initial vol.	500 mL	0	0
After reaction is complete	0	500*x* mL	$\frac{y}{2}\times500$ mL

Now, $500x = 2500 \Rightarrow x = 5$

$\therefore \quad \frac{500y}{2} = 3000 \Rightarrow y = 12$

Thus, the formula of alkane is C_5H_{12}.

73. (b) : Amongst nitrobenzene, aniline, aniline hydrochloride and *N*-acetylaniline only aniline has electron donating $-NH_2$ group. As a result, electron density on the benzene ring increases and hence it becomes reactive towards electrophilic substitution reactions.

74. (c): Antihistamines are vascodialators and antiallergics.

75. (a)

76. (c): Activated charcoal is used for adsorbing poisonous gases.

77. (c): The *cis*-polyisoprene chains are held together by van der Waals forces and has a coiled structure. Thus, it can be stretched like a spring and exhibits elastic properties.

78. (b)

79. (a) : When $\Delta G_{sys} = 0$, the system is in equilibrium.

80. (d) : Proline has a 2° amino group. Hence, it is not estimated by this method.

81.	**(b)**	**82.**	**(d)**	**83.**	**(a)**	**84.**	**(b)**	**85.**	**(d)**
86.	**(a)**	**87.**	**(a)**	**88.**	**(b)**	**89.**	**(d)**	**90.**	**(c)**
91.	**(d)**	**92.**	**(d)**	**93.**	**(a)**	**94.**	**(c)**	**95.**	**(c)**

96. (b) : D - violet, E - indigo, F - red, G - orange.
From last statement, we get that G-orange, E-indigo. As D is able to see green, blue not violet and F can see violet and yellow but not red, so we can say A, B, C have yellow, green, blue and F can't see red only if F wears red and G wears orange, so F must be wearing red.

97. (a)

98. (d) : Triangle shifts center, left, right and so on in bottom row, circle shifts right, center, left and so on in top row and cross shifts center, right, left in middle row.

99. (c) : Upper figure moves to intersect lower figure and bottom line from lower figure is missing.

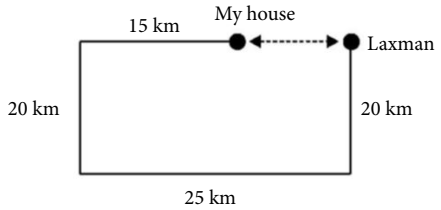
100. (b) :

C	Y	S	T	E	and	B	U	R	P
↓	↓	↓	↓	↓		↓	↓	↓	↓
2	3	4	9	6		1	5	7	8

$\therefore$ 85367 = PUYER.

101. (d) : The two dice shows 2 and 3 in both the faces. So, whenever there are two same faces the other faces are opposite. Hence, 4 and 5 are opposite faces.

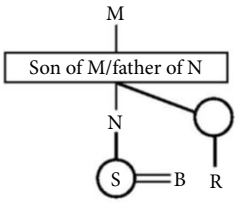
102. (b) :



103. (c)

104. (a) : $(5 \times 2) - 1 = 9$; $(9 \times 2) - 2 = 16$; $(16 \times 2) - 3 = 29$; $(29 \times 2) - 4 = 54$.

105. (d) :



We are not sure about the gender of M. So, data is inadequate to answer this.

106. (a) : Let the coin be tossed n times
Given, the probability of getting at least one head ≥ 0.99
$\therefore$ Probability of getting no head ≤ 0.01

$\Rightarrow \frac{1}{2^n} \leq \frac{1}{100} \Rightarrow 2^n \geq 100 \Rightarrow n = 7.$

107. (a) : Putting $y = x\sqrt{3}$ in the equation of curve

$x^3(3\sqrt{3}+1) + x^2(3\sqrt{3}+14) + (4+5\sqrt{3})x - 1 = 0$

If the roots of this equation are x_1, x_2 and x_3

$\Rightarrow \quad x_1x_2x_3 = \frac{1}{3\sqrt{3}+1} = \frac{3\sqrt{3}-1}{26}$

Also, for a point A, coordinates are $(x_1, x_1\sqrt{3})$

$\therefore \quad OA = \sqrt{x_1^2 + 3x_1^2} = 2x_1$

$\Rightarrow \quad OA \cdot OB \cdot OC = (2x_1)(2x_2)(2x_3) = 8x_1x_2x_3 = \frac{4}{13}(3\sqrt{3}-1)$

108. (c) : Given, $2\log_2 a + 4\log_2 a + 6\log_2 a + \ldots + 40\log_2 a = 840$

$= \log_2 a \{2 + 4 + \ldots + 40\} = 840$

$\Rightarrow \quad \frac{20}{2}(4 + 38)\log_2 a = 420 \log_2 a$

$\Rightarrow \quad 420 \log_2 a = 840 \Rightarrow \log_2 a = 2 \Rightarrow a = 4.$

109. (d) : Assume $x = n + R$, $n \in R$ and R is a fraction < 1

$\therefore \quad L = \lim_{n\to\infty} \frac{\log n}{n+R} \left(\frac{\infty}{\infty}\right)$

Apply L' Hospital's Rule, $L = \lim_{n\to\infty} \left(\frac{1/n}{1}\right) = 0$

110. (a) : The series $1 + 4 + 6 + 5 + 11 + 6 + \ldots$ is a association of two AP's.
$\therefore$ The terms of 1st AP are $(1 + 6 + 11 + \ldots)$ and 2nd AP are $(4 + 5 + 6 + \ldots)$
Sum = (1 + 6 + 11 + ... up to 106 terms) + (4 + 5 + 6 + ... upto 106 terms)

$= \frac{106}{2}(2 \times 1 + 105 \times 5) + \frac{106}{2}(2 \times 4 + 105 \times 1)$

$= \frac{106}{2}[2(5) + 105 \times 6] = 53 \times 640 = 33920$

111. (d) : Given lines are

$L_1 \equiv \vec{r} = (4\hat{i} - 7\hat{j} - 9\hat{k}) + t(3\hat{i} - 7\hat{j} + 4\hat{k})$

$L_2 \equiv \vec{r} = (7\hat{i} - 14\hat{j} - 5\hat{k}) - s(3\hat{i} - 7\hat{j} + 4\hat{k})$

Comparing these with $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$, we get
$\vec{b}=\vec{b}_1=\vec{b}_2=3\hat{i}-7\hat{j}+4\hat{k}$
So both lines are parallel.

$\therefore$ Distance between the lines $=\left|\dfrac{(\vec{a}_1-\vec{a}_2)\times\vec{b}}{|\vec{b}|}\right|$

$=\left|\dfrac{(3\hat{i}-7\hat{j}+4\hat{k})\times(3\hat{i}-7\hat{j}+4\hat{k})}{|3\vec{i}-7\vec{j}+4\vec{k}|}\right|=0$

112. (d) : By cosine rule

$$\frac{2(b^2+c^2-a^2)+(c^2+a^2-b^2)+2(a^2+b^2-c^2)}{2abc}=\frac{a^2+b^2}{abc}$$

$\Rightarrow$ $3b^2 + c^2 + a^2 = 2a^2 + 2b^2$ $\Rightarrow$ $a^2 = b^2 + c^2$

$\Rightarrow$ $A = 90°$

113. (c) : If α, β, γ are roots of $x^3 + ax^2 + bx + c = 0$, then

$$(\alpha+\beta)^{-1}+(\beta+\gamma)^{-1}+(\gamma+\alpha)^{-1}=\frac{a^2+b}{-ab+c}=\frac{a^2+b}{c-ab}$$

Now in the given problem, we have $a = 0$, $b = 4$, $c = 1$

$$\Rightarrow \quad \frac{a^2+b}{c-ab}=\frac{4}{1}$$

114. (d) : $f'(x)=A\dfrac{\pi}{2}\cos\left(\dfrac{\pi x}{2}\right)$

$$\Rightarrow \quad \sqrt{2}=f'(1/2)=A\frac{\pi}{2}\left(\frac{1}{\sqrt{2}}\right) \quad \Rightarrow \quad A=\frac{4}{\pi}$$

Now, $\displaystyle\int_0^1 f(x)dx=\frac{2A}{\pi}$ $\Rightarrow$ $\dfrac{2A}{\pi}=\dfrac{4}{\pi}\displaystyle\int_0^1\sin\left(\frac{\pi}{2}x\right)dx+\int_0^1 B dx$

$$\Rightarrow \quad \frac{2A}{\pi}=-\frac{4}{\pi}\frac{\cos(\pi x/2)}{\pi/2}\Big|_0^1+B \Rightarrow \frac{2A}{\pi}=\frac{4}{\pi}\times\frac{2}{\pi}+B \Rightarrow B=0$$

115. (b) : Given $yy'=\dfrac{y}{y'}$ $\Rightarrow$ $y'^2=1$

Length of tangent is $\dfrac{y}{y'}\sqrt{1+(y')^2}=|\sqrt{2}y|$

116. (d) : Given $\cos^{-1}x-\cos^{-1}\dfrac{y}{2}=\alpha$

$$\Rightarrow \quad \frac{xy}{2}+\sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}}=\cos\alpha$$

$$\Rightarrow \quad \left(\cos\alpha-\frac{xy}{2}\right)^2=(1-x^2)\left(1-\frac{y^2}{4}\right)$$

$\Rightarrow$ $4x^2 - 4xy\cos\alpha + y^2 = 4(1 - \cos^2\alpha) = 4\sin^2\alpha$.

117. (b) : Q be the image of the point $P(2, 3, 4)$ in the plane $x - 2y + 5z = 6$, then PQ is normal to the plane.

$\therefore$ Direction ratios of PQ are $< 1, -2, 5 >$

Since PQ passes through $P(2, 3, 4)$ and has direction ratios $1, -2, 5$

$\therefore$ Equation of line PQ is $\dfrac{(x-2)}{1}=\dfrac{y-3}{-2}=\dfrac{z-4}{5}$

118. (c) : $\dfrac{{}^nC_{r+1}}{{}^nC_r}=\dfrac{56}{28}$ $\Rightarrow$ $\dfrac{n-r}{r+1}=2$ $\Rightarrow$ $n=3r+2$

Similarly, $\dfrac{{}^nC_{r+2}}{{}^nC_{r+1}}=\dfrac{70}{56}$ $\Rightarrow$ $\dfrac{n-(r+1)}{r+2}=\dfrac{5}{4}$

$$\Rightarrow \quad \frac{(3r+2)-(r+1)}{r+2}=\frac{5}{4} \quad \Rightarrow \quad \frac{2r+1}{r+2}=\frac{5}{4}.$$

$\Rightarrow$ $8r + 4 = 5r + 10$ $\Rightarrow$ $3r = 6$ $\Rightarrow$ $r = 2$

Now, $n = 3r + 2$ $\Rightarrow$ $n = 3(2) + 2 = 8$

119. (a) : Given $f(x) = x^{1/x}$, $x > 0$

Since $x = e$ is a point of maxima.

$\therefore$ $f(e) > f(x)$; $x > 0$

$\Rightarrow$ $f(e) > f(\pi)$ (in particular) $\Rightarrow$ $e^{1/e} > \pi^{1/\pi} \Rightarrow e^{\pi} > \pi^e$

$\therefore$ The required condition is $e^{\pi} > \pi^e$.

120. (b) : $f(x)=\dfrac{\text{cosec}^{-1}x+\sec^{-1}x}{\text{cosec}\,x}$ [$\because \cos^{-1}(1/x) = \sec^{-1}x$]

$\Rightarrow$ $f(x)=\dfrac{\pi}{2}\sin x$

Now, $-1 \le \sin x \le 1$

$\therefore$ $-\dfrac{\pi}{2}\le\dfrac{\pi}{2}\sin x\le\dfrac{\pi}{2}$ $\Rightarrow -\dfrac{\pi}{2}\le f(x)\le\dfrac{\pi}{2}$

121. (b) : The five persons who refuse to sit in the upper deck must be accommodated in the lower deck and the eight refusing to travel in the lower deck must be accommodated in the upper deck. Thus there will be only five vacant seats in the upper deck and only two in the lower deck. These vacant seats are to be filled by the remaining seven passengers and this can be done in ${}^7C_5 \cdot {}^2C_2 = 21$ ways.

122. (a) **123. (d)**

124. (d) : If X_1 and X_2 are optimal solutions of an L.P.P., then $\lambda X_1 + (1 - \lambda)X_2$, $0 \le \lambda \le 1$ is also an optimal solution.

125. (d) : $\tan n\pi = 0\ \forall\, n \in Z$ $\therefore$ $f(x) = 0\ \forall x$, a constant function. $\therefore$ $f''(x) = 0\ \forall x$.

126. (c) : Let E be the event that engineering subject is selected and A, B be the events that group I and II is selected respectively.

Now, $P(A) = 2/6$, $P(B) = 4/6$

$\therefore$ $P(E) = P(A) \times P(E/A) + P(B) \times P(E/B)$

$=\dfrac{2}{6}\cdot\dfrac{3}{8}+\dfrac{4}{6}\cdot\dfrac{5}{8}=\dfrac{13}{24}$

127. (d) : $\because$ aRb if $\sin^2 a + \cos^2 b = 1$

$\because$ $\sin^2 a + \cos^2 a = 1$ for all $a \in R$

$\therefore$ aRa $\Rightarrow$ R is reflexive

$\because$ $\sin^2 a + \cos^2 b = 1$ $\Rightarrow$ $1 - \cos^2 a + 1 - \sin^2 b = 1$

$\Rightarrow$ $\sin^2 b + \cos^2 a = 1$ $\Rightarrow$ bRa

$\therefore$ R is symmetric

Further, let aRb and bRc

i.e. $\sin^2 a + \cos^2 b = 1$ … (i)

and $\sin^2 b + \cos^2 c = 1$ … (ii)

Adding, $\sin^2 a + 1 + \cos^2 c = 2$

$\Rightarrow$ $\sin^2 a + \cos^2 c = 1$ $\Rightarrow$ aRc

$\therefore$ R is also transitive.

$\therefore$ R is an equivalence relation.

128. (d) : Given,

$$5\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)+7\sin^{-1}\left(\frac{2x}{1+x^2}\right)-4\tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$- \tan^{-1}(x) = 5\pi$

Put $x = \tan\theta$

$\Rightarrow \quad 5\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)+7\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$$-4\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)-\tan^{-1}(\tan\theta)=5\pi$$

$\Rightarrow \quad 5 \times 2\theta + 7 \times 2\theta - 4 \times 2\theta - \theta = 5\pi$

$\Rightarrow \quad 10\theta + 14\theta - 9\theta = 5\pi \quad \Rightarrow 15\theta = 5\pi \quad \Rightarrow \quad \theta = \pi/3$

$\therefore \; x = \tan\theta = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

129. (b) : Let (x, y) be the locus of centre.

$\therefore \quad (x, y) = \frac{(3,3)+(x_0, 4-x_0)}{2}$,

where x_0, y_0 lies on $x + y = 4$.

$\Rightarrow \quad x = \frac{3+x_0}{2}; \; y = \frac{3+4-x_0}{2}$

$\therefore \quad 2x = 3 + x_0; \; 2y = 7 - x_0$

Adding equations, we get

$2x + 2y = 10 \Rightarrow x + y = 5$ is the locus.

$\therefore \quad k = 5$

130. (a) : There are 4 sets of 3 identical books. The desired number is $\frac{12!}{(3!)^4} = 369600$

131. (c) : $[\vec{u}\;\vec{v}\;\vec{w}] = \vec{u}\cdot(\vec{v}\times\vec{w}) \le |\vec{u}||\vec{v}\times\vec{w}| = |\vec{v}\times\vec{w}|$

But $\vec{v}\times\vec{w} = 3\hat{i} - 7\hat{j} - \hat{k} \Rightarrow |\vec{v}\times\vec{w}| = \sqrt{59}$

$\therefore \quad [\vec{u}\;\vec{v}\;\vec{w}] \le \sqrt{59}$

132. (c) : $y = (1 + x)(1 + x^2)(1 + x^4) \ldots\ldots (1 + x^{2n})$

$\Rightarrow \quad \log y = \log(1 + x) + \log(1 + x^2) + \log(1 + x^4) + \ldots\ldots$

$$+ \log(1 + x^{2n})$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x} + \frac{1\times 2x}{1+x^2} + \frac{4x^3}{1+x^4} + \ldots\ldots + \frac{2nx^{2n-1}}{1+x^{2n}}$$

$$\therefore \quad \frac{dy}{dx} = y\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \ldots\ldots + \frac{2nx^{2n-1}}{1+x^{2n}}\right]$$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{x=0} = (y)_{x=0}[1] = 1\cdot 1 = 1$$

133. (b) : $a = \sqrt{2011^2 + 2012} = 2011.5$

$f(a) = [2011.5] = 2011.$

134. (b) : $\because \quad f(x) = \frac{2-1}{\lfloor 1}x + \frac{2^2-1}{\lfloor 2}x^2 + \frac{2^3-1}{\lfloor 3}x^3 + \ldots\ldots$

$$= \left(\frac{2x}{\lfloor 1} + \frac{(2x)^2}{\lfloor 2} + \frac{(2x)^3}{\lfloor 3} + \ldots\right) - \left(\frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \ldots\right)$$

$$= \left[1 + \frac{2x}{\lfloor 1} + \frac{(2x)^2}{\lfloor 2} + \frac{(2x)^3}{\lfloor 3} + \ldots.\right] - \left[1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \ldots\right]$$

$= e^{2x} - e^x$

$\therefore \quad f(x) = 0 \quad \Rightarrow \quad e^x(e^x - 1) = 0$

$\Rightarrow \quad e^x - 1 = 0 \qquad [\because \; e^x \ne 0]$

$\Rightarrow \quad x = 0$ is the only real solution.

135. (b) : $\frac{a^2-b^2}{ab}\tan\left\{i(\ln(a-ib) - \ln(a+ib))\right\}$

$= \frac{a^2-b^2}{ab}\tan\left\{i(\ln re^{-i\theta} - \ln re^{i\theta})\right\}$, where $\theta = \tan^{-1}\frac{b}{a}$

and $r = \sqrt{a^2+b^2}$

$= \frac{a^2-b^2}{ab}\tan\{-2i \times i\theta\} = \frac{a^2-b^2}{ab} \times \tan 2\theta$

$= \frac{a^2-b^2}{ab} \times \frac{2ab}{a^2-b^2} = 2$

136. (d) : $(1 - x + x^2)^n = a_0 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n}$...(i)

Putting $x = 1$ in (i), we get

$a_0 + a_1 + a_2 + \ldots + a_{2n} = 1$ ...(ii)

Putting $x = -1$ in (i), we get

$a_0 - a_1 + a_2 - \ldots + a_{2n} = 3^n$ ...(iii)

Adding (ii) & (iii), we get

$2(a_0 + a_2 + a_4 + \ldots\ldots + a_{2n}) = 1 + 3^n$

$$\Rightarrow \quad a_0 + a_2 + a_4 + \ldots\ldots + a_{2n} = \frac{1+3^n}{2}$$

137. (b) : $A(A + I) = -4I$

$|A||A + I| = 4$

Both A and $A + I$ are non-singular.

138. (b) : $x^2 + (k - 1)x + 9 > 0 \;\; \forall \;\; x$

$\Delta < 0 \Rightarrow (k - 1)^2 - 36 < 0 \quad \Rightarrow \quad (k - 1 + 6)(k - 1 - 6) < 0$

$(k + 5)(k - 7) < 0 \Rightarrow k \in (-5, 7).$

139. (b) : $0 \le x \le \frac{\pi}{2} \quad \therefore \quad \tan x > 0, \cot x > 0$

$$f(x) = \frac{ab(a^2-b^2)\sin x\cos x}{a^2\sin^2 x + b^2\cos^2 x} = \frac{ab(a^2-b^2)}{a^2\tan x + b^2\cot x}$$

$$= \frac{ab(a^2-b^2)}{(a\sqrt{\tan x} - b\sqrt{\cot x})^2 + 2ab}$$

$f(x)$ will be maximum when $(a\sqrt{\tan x} - b\sqrt{\cot x})^2$ is minimum.

But its minimum value is zero.

$\therefore \quad$ Max. value of $f(x) = \frac{ab(a^2-b^2)}{2ab} = \frac{a^2-b^2}{2}$

140. (b) : Given determinant shows the volume of parallelepiped made by vectors $\vec{a}, \vec{b}$ and $\vec{c}$

$$\therefore \quad \begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix} = \vec{a}\cdot(\vec{b}\times\vec{c}) = |\vec{a}||\vec{b}\times\vec{c}|\cos\frac{\pi}{6}$$

$$= |\vec{a}||\vec{b}\times\vec{c}|\sin\frac{\pi}{3} = |\vec{a}||\vec{b}||\vec{c}|\sin\frac{\pi}{3}\sin\frac{\pi}{6}$$

$$= 2\sqrt{2}\,|\vec{b}||\vec{c}|\frac{\sqrt{3}}{2}\times\frac{1}{2} = \frac{\sqrt{3}\,|\vec{b}||\vec{c}|}{\sqrt{2}}$$

$$\therefore \quad \begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix}^n = \left(\frac{\sqrt{3}|\vec{b}||\vec{c}|}{\sqrt{2}}\right)^n$$

141. (b) : $\int \frac{2e^x+3e^{-x}}{3e^x+4e^{-x}}dx = \int \frac{2e^{2x}+3}{3e^{2x}+4}dx$

$= \int \frac{2e^{2x}}{3e^{2x}+4}dx + \int \frac{3dx}{3e^{2x}+4} = \frac{2}{6}\int \frac{6e^{2x}}{3e^{2x}+4}dx + 3\int \frac{e^{-2x}}{3+4e^{-2x}}dx$

$= \frac{2}{6}\int \frac{6e^{2x}}{3e^{2x}+4}dx + \left(\frac{3}{-8}\right)\int \frac{-8e^{-2x}}{3+4e^{-2x}}dx$

$= \frac{1}{3}\log(3e^{2x}+4) + \left(\frac{3}{-8}\right)\log\left(\frac{3e^{2x}+4}{e^{2x}}\right)$

$= \frac{1}{3}\log(3e^{2x}+4) + \left(\frac{3}{-8}\right)\log(3e^{2x}+4) + \left(\frac{-3}{8}\right)(-2x)$

$= \frac{3}{4}x - \frac{1}{24}\log(3e^{2x}+4) = Ax + B\log(3e^{2x}+4)$

$\therefore \quad A = 3/4, B = -1/24$

142. (a) : $(X-x)\left(\frac{\partial f}{\partial x}\right) + (Y-y)\left(\frac{\partial f}{\partial y}\right) = 0$

$(X - x)(4x^3) + (Y - y)4y^3 = 0$

or $Xx^3 + Yy^3 = x^4 + y^4 = a^4$ ...(i)

If intercepts are p and q, then $(p, 0)$ and $(0, q)$ satisfy (i)

$\therefore \quad px^3 = a^4, \; qy^3 = a^4$

$p^{-4/3} + q^{-4/3} = (a^4)^{-4/3}(x^4 + y^4) = a^{-4/3}$

143. (c) : Given, $\begin{bmatrix} 1+\omega+a & 2\omega-\omega \\ -2\omega+3\omega & 2-b \end{bmatrix} = \begin{bmatrix} 0 & \omega \\ \omega & 1 \end{bmatrix}$

$= \begin{vmatrix} 1+\omega+a & \omega \\ \omega & 2-b \end{vmatrix}$

On comparing, we get $a = \omega^2, b = 1$ $(\because 1 + \omega + \omega^2 = 0)$

$\therefore \quad a^2 + b^2 = (\omega^2)^2 + (1)^2 = 1 + \omega$ $(\because \omega^3 = 1)$

144. (c) : Given $\int_1^b f(x)dx = (b-1)\sin(3b+4)$

Differentiating both sides w.r.t. b, we get

$f(b) = 3(b - 1)\cos(3b + 4) + \sin(3b + 4)$

$\therefore \quad f(x) = 3(x - 1)\cos(3x + 4) + \sin(3x + 4)$

145. (b) : The equation of normal at $P(6, 3)$ is

$\frac{a^2x}{6} - \frac{b^2y}{3} = a^2 + b^2$

As it passes through $(9, 0)$, we have $\frac{9a^2}{6} = a^2 + b^2$

$\Rightarrow \frac{a^2}{2} = b^2 \Rightarrow \frac{b^2}{a^2} = \frac{1}{2}$. Thus $e = \sqrt{1+\frac{1}{2}} = \sqrt{\frac{3}{2}}$.

146. (a) : The feasible region is always a convex region.

147. (b) : $(x + y)\,dy + (y - x)dx = 0$

$\Rightarrow \quad ydy + xdy + ydx - xdx = 0 \Rightarrow y^2 + 2xy - x^2 = c$

$\Rightarrow \quad y(1) = 1 \Rightarrow c = 2 \Rightarrow (y(0))^2 = 2$

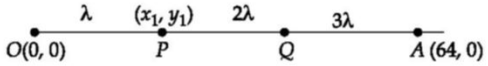
148. (a) : $\Sigma a^2\, 2R\sin A\sin(B - C)$

$= \Sigma a^2\, 2R\sin(\pi - B + C)\sin(B - C)$

$= 2R\,\Sigma a^2 \sin(B + C)\sin(B - C) = 2R\,\Sigma a^2(\sin^2 B - \sin^2 C)$

$= 2R\,\Sigma a^2\left\{\left(\frac{b}{2R}\right)^2 - \left(\frac{c}{2R}\right)^2\right\} = \frac{1}{2R}\Sigma a^2(b^2 - c^2) = 0$

149. (a) :



P divides OA in the ratio $1 : 5$

$x_1 = \frac{1\times 64 + 5\times 0}{1+5} = \frac{64}{6} = \frac{32}{3}$

$y_1 = \frac{1\times 0 + 5\times 0}{1+5} = 0$. So, point P is $\left(\frac{32}{3}, 0\right)$

150. (c) : We have, $f(x) = \frac{x^2+e}{x^2+1}$

$f'(x) = \frac{2x(x^2+1) - 2x(x^2+e)}{(x^2+1)^2} = \frac{2x^3 + 2x - 2x^3 - 2ex}{(x^2+1)^2}$

$= \frac{2x - 2xe}{(x^2+1)^2} = \frac{2x(1-e)}{(x^2+1)^2} < 0$

$f'(x) < 0$, $f(x)$ is decreasing

Hence, f is one-one function.

$x \to 0$, $f(x) \to e$ and $x \to \infty$, $f(x) \to 1$

$\therefore \quad$ Range $= (1, e)$ = co-domain.

Hence, f is onto function.

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OLYMPIAD CORNER

1. (a) Show that there is a positive integer n so that the interval $\left(\left(n+\frac{1}{1994}\right)^2, \left(n+\frac{1}{1993}\right)^2\right)$ contains an integer N.

(b) Find the smallest integer N which is contained in such an interval for some n.

2. Let a, b, c be real numbers. Their pairwise sum $a+b$, $b+c$ and $c+a$ are written on three round cards and their pairwise products ab, bc and ca are written on three square cards. We call (a, b, c) a tadpole if we can form three pairs of cards, each consisting of one round card and one square card with the same number on both. An example of a tadpole is $(0, 0, 0)$.

(a) Find all possible tadpoles of the form (a, a, a).

(b) Prove that there is a tadpole that is not of the form (a, a, a). (you do not have to find the actual values of a, b and c.)

3. Let ABC be a (variable) isosceles triangle with constant sides $a = b$ and variable side c. Denote the median, angle bisector and altitude, measured from A to the opposite side, by m, w and h respectively. Find $\lim\limits_{c\to a}\dfrac{m-h}{w-h}$.

4. Prove that for all positive real numbers a, b, c the following inequality holds:

$$2\sqrt{bc+ca+ab} \le \sqrt{3}\sqrt[3]{(b+c)(c+a)(a+b)}.$$

5. Let k be a natural number and u, v be real numbers. Set $P(x) = (x-u^k)(x-uv)(x-v^k) = x^3 + ax^2 + bx + c$.

(a) For $k = 2$ prove: If a, b, c are rational, then the product uv is rational.

(b) Is that also true for $k = 3$?

6. For natural $n \ge 1$, let $s(n)$ denote the sum of all positive divisors of n. Prove that for every integer $n > 1$ the product $s(n-1)s(n)s(n+1)$ is even.

7. Given a circle k of centre M and radius r, let AB be a fixed diameter of k and let K be a fixed point of segment AM. Denote by t the line tangent to k at A. For any chord CD (other than AB) passing through K construct P and Q as the points of intersection of lines BC and BD with t. Prove that the product $AP.AQ$ remains constant as the chord CD varies.

SOLUTIONS

1. Since $\left(n+\frac{1}{1994}\right)^2 = n^2 + \frac{2n}{1994} + \frac{1}{(1994)^2}$

and $\left(n+\frac{1}{1993}\right)^2 = n^2 + \frac{2n}{1993} + \frac{1}{(1993)^2}$,

For minimality we want

$$\frac{2n}{1994} + \frac{1}{1994^2} < k < \frac{2n}{1993} + \frac{1}{1993^2}$$

for some positive integer k, with k minimal. When $k = 1$ we have

$$\left(1-\frac{1}{1993^2}\right)996\frac{1}{2} < n < \left(1-\frac{1}{1994^2}\right)997$$

i.e., $996\frac{1}{2} - \delta < n < 997 - \varepsilon$ for some small positive reals δ and ε. This gives $n < 997$ and $n > 996$, for which no such integer exists.

However, for $k = 2$, we get

$$996\frac{1}{2} - \delta + 996\frac{1}{2} < n < 997 - \varepsilon + 997.$$

So $1993 - \delta < n < 1994 - \varepsilon$

so that $n = 1993$ and $N_{\min} = (1993)^2 + 2$.

2. (a) If $b = c = a$, there are 3 pairs of cards $2a$ and a^2. Now, $a^2 = 2a$ iff $a = 0$ or $a = 2$. Thus there are exactly two tadpoles of the form (a, a, a), namely $(0, 0, 0)$ and $(2, 2, 2)$.

(b) **Case I.** Two of a, b, c are equal: say a, a, b $(b \ne a)$. This leads to pairings

(i) $(2a, a^2)$, $(a + b, ab)$ and $(a + b, ab)$.
The first pair of which implies $a = 0$ or 2
$a = 0 \Rightarrow ab = 0 \Rightarrow a + b = 0 \Rightarrow b = 0 = a$,
$a = 2 \Rightarrow b + 2 = 2b \Rightarrow b = 2 = a$,
(ii) $(2a, ab)$, $(a + b, a^2)$ and $(a + b, ab)$. Here, the last two pairs give $a^2 = ab$ implying $a = 0$ or $a = b$.
Now, $a = 0 \Rightarrow b = 0 = a$.
Case II. a, b, c are all distinct. [Throughout, the assumption is made implicitly that a, b, c do not take values which would render any of the others unbounded].
(i) $(a + b, ab)$, $(b + c, bc)$ and $(c + a, ca)$.

$ab = a + b \Rightarrow (a - 1)(b - 1) = 1$ and $b = 1 + \dfrac{1}{a-1}$.
Similarly

$$c = \frac{1}{1+\dfrac{1}{b-1}} \text{ and } a = 1 + \frac{1}{c-1} = 1 + \frac{1}{1+\dfrac{1}{b-1}} = b.$$

(ii) $(a + b, bc)$, $(b + c, ab)$, $(c + a, ac)$.
Now $a + b = bc \Rightarrow c = 1 + 1/b$. Then $b + 1 + a/b = ab$ $\Rightarrow b + 1 = a(b - 1/b) \Rightarrow a = b/(b - 1)$ or $b = -1$.

$$a = \frac{b}{b-1} \Rightarrow c = 1 + \frac{1}{b-1} = \frac{b}{b-1} = a.$$

$b = -1 \Rightarrow a^2 - a + 1 = 0$ and a is not real.
(iii) $(a + b, ca)$, $(c + a, ab)$ and $(c + a, bc)$.
Now
$$\left.\begin{aligned} a+b=ca &\Rightarrow b = a(c-1) \\ b+c=ab &\Rightarrow b = \frac{c}{a-1} \end{aligned}\right\} \Rightarrow a(a-1) = \frac{c}{c-1}.$$

Also $c + a = ac(c - 1) \Rightarrow a = \dfrac{c}{c^2 - c - 1}$.

so that $\left(\dfrac{c}{c^2-c-1}\right)\left(\dfrac{2c-c^2+1}{c^2-c-1}\right) = \dfrac{c}{c-1}$.

This gives $c^4 - c^3 - 4c^2 + 3c + 2 = 0$
$\Rightarrow (c - 2)(c^3 + c^2 - 2c - 1) = 0$.
Now $c = 2$ gives $a^2 - a - 2 = 0 \Rightarrow a = 2$, a contradiction, or $a = -1 \Rightarrow b = -1$, a contradiction.
Thus $c^3 + c^2 - 2c + 1 = 0$ is required to have a real solution, which it must do since complex roots occur in conjugate pairs. (Note only (iii) is necesssary in order to establish the existence of solutions, provided it is now checked that $c - 1 \neq 0$ and $c^2 - c - 1 \neq 0$. The other cases are included for completeness.)
In fact, using Cardan's method and complex numbers, the three real roots of the cubic $c^3 + c^2 - 2c - 1 = 0$ can be found. They are

$$c_1 = \frac{2\sqrt{7}}{3}\cos\frac{\theta}{3} = \frac{1}{3}, \quad c_2 = \frac{2\sqrt{7}}{3}\cos\left(\frac{2\pi-\theta}{3}\right) - \frac{1}{3}$$

and $c_3 = \dfrac{2\sqrt{7}}{3}\cos\left(\dfrac{2\pi+\theta}{3}\right) - \dfrac{1}{3}$, where $\theta = \cos^{-1}\left(\dfrac{1}{\sqrt{28}}\right)$.
There are then 6 tadpoles not of the form (a, a, a) being the six permutations of (c_1, c_2, c_3).

3. Note first that

$$\lim_{c\to a}\frac{m-h}{w-h} = \lim_{c\to a}\left(\frac{m^2-h^2}{w^2-h^2}\cdot\frac{w+h}{m+h}\right) = \lim_{c\to a}\frac{m^2-h^2}{w^2-h^2},$$

since $\lim\limits_{c\to a}\dfrac{m+h}{w+h} = 1$
(if $a = b = c$ then $w = m = h$). Now

$$m^2 - h^2 = \frac{2(b^2+c^2)-a^2}{4} - \frac{4}{a^2}s(s-a)(s-b)(s-c)$$

(where s is the semiperimeter of the triangle) and since $a = b$ we have

$$m^2 - h^2 = \frac{a^2+2c^2}{4} - \frac{4}{a^2}\left(a+\frac{c}{2}\right)\cdot\frac{c}{2}\cdot\frac{c}{2}\left(a-\frac{c}{2}\right)$$
$$= \frac{a^2+2c^2}{4} - \frac{c^2}{4a^2}(4a^2-c^2) = \frac{(a^2-c^2)^2}{4a^2}.$$

Also $w^2 - h^2 = \dfrac{4bc}{(b+c)^2}s(s-a) - \dfrac{4}{a^2}s(s-a)(s-b)(s-c)$
and putting $a = b$ we easily find that

$$w^2 - h^2 = \frac{4ac}{(a+c)^2}\left(a+\frac{c}{2}\right)\frac{c}{2} - \frac{c^2}{4a^2}(4a^2-c^2)$$
$$= \frac{c^2(2a+c)}{4a^2(a+c)^2}[4a^3 - (2a-c)(a+c)^2] = \frac{c^2(2a+c)^2(a-c)^2}{4a^2(a+c)^2}.$$

Thus $\dfrac{m^2-h^2}{w^2-h^2} = \dfrac{(a+c)^4}{c^2(2a+c)^2}$,

so $\lim\limits_{c\to a}\dfrac{m-h}{w-h} = \lim\limits_{c\to a}\dfrac{m^2-h^2}{w^2-h^2} = \dfrac{(2a)^4}{a^2(3a)^2} = \dfrac{16}{9}$.

4. It is enough to prove that for all positive real numbers a, b, c the following inequality holds:
$64(bc + ca + ab)^3 \leq 27(a + b)^2(a + c)^2(b + c)^2$
or $64 \cdot 3(bc + ca + ab)(bc + ca + ab)^2 \leq 81[(a + b)(a + c)(b + c)]^2$.
It is known that $3(bc + ca + ab) \leq (a + b + c)^2$. Thus, it is enough to prove one of the following equivalent inequalities:

MPP CLASS XII ANSWER KEY

1.	(b)	**2.**	(a)	**3.**	(b)	**4.**	(d)	**5.**	(a)
6.	(b)	**7.**	(a,c)	**8.**	(b,c)	**9.**	(a,b,c,d)		
10.	(a,b)	**11.**	(a,c)	**12.**	(a,b,c,d)			**13.**	(c)
14.	(c)	**15.**	(c)	**16.**	(c)	**17.**	(4)	**18.**	(6)
19.	(4)	**20.**	(14)						

$8(a+b+c)(bc+ca+ab) \le 9(a+b)(a+c)(b+c)$
$8(a+b)(a+c)(b+c) + 8abc \le 9(a+b)(a+c)(b+c)$
$8abc \le (a+b)(a+c)(b+c)$
which is well known to hold.

5. Expanding $P(x)$ we find that
$a = -(u^k + uv + v^k)$, $b = (uv)(u^k + u^{k-1}v^{k-1} + v^k)$,
$c = -u^{k+1}v^{k+1}$.
(a) For $k = 2$, we have $a = -(u^2 + uv + v^2)$,
$b = uv(u^2 + uv + v^2)$. If $u = v = 0$ then uv is rational. Otherwise

$$a = -\left[\left(u+\frac{v}{2}\right)^2 + \frac{3}{4}v^2\right] \neq 0$$

and thus $uv = -b/a$ which is rational.
(b) For $k = 3$ the implication need not be true; for example, if $u = -\sqrt[3]{2}$ and $v = \sqrt[6]{2}$, then $uv = -\sqrt{2}$ while

$$a = -(u^3 + uv + v^3) = -(-2 - \sqrt{2} + \sqrt{2}) = 2,$$
$$b = uv(u^3 + u^2v^2 + v^3) = -\sqrt{2}(-2 + 2 + \sqrt{2}) = -2$$

and $c = -u^4v^4 = -4$ are all rational.

6. We show equivalently that at least one of $s(n-1)$, $s(n)$ and $s(n+1)$ must be even. Let $n = P_1^{t_1} P_2^{t_2} \ldots P_k^{t_k}$ be the prime powers decomposition of n where the P_i's are distinct primes. Then it is well known that

$$s(n) = \prod_{i=1}^{k} (1 + P_i + P_i^2 + \ldots + P_i^{t_i}).$$

If $P_i = 2$, then the factor $p_i = 1 + P_i^2 + \ldots + P_i^{t_i}$ is clearly odd. If P_i is odd, then clearly p_i is odd iff t_i is even. Hence $s(n)$ is odd if and only if $n = 2^d q^2$, where $d \ge 0$ and q is odd. Suppose, contrary to what we claim, that $s(n-1)$, $s(n)$ and $s(n+1)$ are all odd. Then $n - 1 = 2^a q_1^2$, $n = 2^b q_2^2$ and $n + 1 = 2^c q_3^2$ where $a, b, c \ge 0$ and $q_i (i = 1, 2, 3)$ are all odd. We consider two cases:
Case (i) : If n is even, then $a = c = 0$. Thus $n - 1 = q_1^2$ and $n + 1 = q_3^2$ which imply $(q_3 - q_1)(q_3 + q_1) = 2$ and thus $q_3 - q_1 = 1$ and $q_3 + q_1 = 2$ which clearly yields no integer solutions, a contradiction.
Case (ii). If n is odd then $b = 0$, $a, c \ge 1$ and $n = q_2^2$. Thus

$$q_2^2 - 2^a q_1^2 = 1 \quad \ldots(i)$$

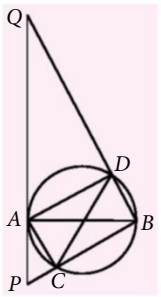
Furthermore $2^c q_3^2 = n + 1 = q_2^2 + 1$. If $c \ge 2$, then $2^c q_3^2 \equiv 0 \pmod 4$ while $q_2^2 + 1 \equiv 2 \pmod 4$ since the square of any odd integer is congruent to 1 (mod 4). Thus $c = 1$ and from $2q_3^2 = 2^1 a_1^2 + 2$ we get

$$q_3^2 - 2^{a-1} q_1^2 = 1 \quad \ldots(ii)$$

Note that equation $x^2 - y^2 = 1$ has no solutions in positive integers since $(x - y)(x + y) = 1$ is clearly impossible. Hence if a is even then (i) is not solvable while if a is odd then (ii) is not solvable. Again we have a contradiction.

7. Because $\angle BAP = 90°$ and $\angle ACB = 90°$ we get ΔAPB similar to ΔCAB. Thus we have $\frac{AP}{AB} = \frac{CA}{CB}$. Similarly we have $\frac{AQ}{AB} = \frac{DA}{DB}$. Multiplying both sides gives

$$\frac{AP \cdot AQ}{AB^2} = \frac{CA \cdot DA}{CB \cdot DB}. \quad \ldots(i)$$



As A, C, B, D are concyclic we get ΔACK similar to ΔDBK and ΔDAK similar to ΔBCK. Thus we have

$$\frac{KA}{BD} = \frac{CK}{BK} \text{ and } \frac{DA}{BC} = \frac{AK}{CK}.$$

Therefore

$$\frac{CA}{BD} \cdot \frac{DA}{BC} = \frac{CK}{BK} \cdot \frac{AK}{CK} = \frac{AK}{BK} \quad \ldots(ii)$$

From (i) and (ii) we get

$$\frac{AP \cdot AQ}{AB^2} = \frac{AK}{BK}.$$

Hence we have $AP \cdot AQ = AB^2 \times \frac{AK}{BK}$, a constant.

SOLUTION SENDERS

Samurai Sudoku (February)

- Mareenamma Devasia
- G. Dilip Raj (Telangana)
- Arijit Dalui (Kolkata)
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Mathdoku (February)

- Ramya Kamath (Karnataka)
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CLASS XII

CBSE DRILL

Practice Paper 2019

Time Allowed : 3 hours | Maximum Marks : 100

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Questions **1-4** in Section-A are very short-answer type questions carrying **1** mark each.
(iv) Questions **5-12** in Section-B are short-answer type questions carrying **2** marks each.
(v) Questions **13-23** in Section-C are long-answer-**I** type questions carrying **4** marks each.
(vi) Questions **24-29** in Section-D are long-answer-**II** type questions carrying **6** marks each.

SECTION - A

1. A die is tossed thrice. Geeting an even number is considered a success. What is the variance of the binomial distribution?

2. Find the order and degree of the differential equation $\frac{d^5y}{dx^5}+e^{dy/dx}+y^2=0$.

OR

Determine the order and degree of differential equation $\frac{d^3x}{dt^3}+\frac{d^2x}{dt^2}+\left(\frac{dx}{dt}\right)^2=e^t$.

3. If $f(x) = 27x^3$ and $g(x) = x^{1/3}$, find $gof(x)$.

4. If $\vec{a}=2\hat{i}+\hat{j}+3\hat{k},\vec{b}=-\hat{i}+2\hat{j}+\hat{k},\vec{c}=-3\hat{i}+\hat{j}+2\hat{k}$, find $[\vec{a}\,\vec{b}\,\vec{c}]$.

SECTION - B

5. If a line makes angles α, β and γ with the coordinate axes, prove that $(\cos 2\alpha + \cos 2\beta, + \cos 2\gamma) = -1$.

6. Find the value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$.

7. Evaluate $P(A \cup B)$, if $2P(A) = P(B) = 6/13$ and $P(A/B) = 1/3$.

OR

If two events A and B are such that $P(\overline{A})=0.3$, $P(B)=0.4$ and $P(A\cap\overline{B})=0.5$, then find $P(B|A\cup\overline{B})$.

8. Write the projection of $(\vec{b}+\vec{c})$ on $\vec{a}$, where $\vec{a}=(2\hat{i}-2\hat{j}+\hat{k})$, $\vec{b}=(\hat{i}+2\hat{j}-2\hat{k})$ and $\vec{c}=(2\hat{i}-\hat{j}+4\hat{k})$.

9. Verify Rolle's Theorem for the function $f(x) = \log(x^2 + 2) - \log 3$ in $[-1, 1]$.

10. Let $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down gof.

OR

The binary operation $* : R \times R \to R$, is defined as $a * b = 2a + b$. Find $(2 * 3) * 4$.

11. Find a matrix X such that $2A + B + X = O$, where $A=\begin{bmatrix}-1 & 2\\ 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}3 & -2\\ 1 & 5\end{bmatrix}$.

12. Evaluate : $\int \frac{(x-1)}{\sqrt{x+4}}dx$

OR

Evaluate : $\int_0^{\pi/2} x\cos x\,dx$

SECTION - C

13. Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$.

14. Find the values of p for which the curves $x^2 = 9p(9 - y)$ and $x^2 = p(y + 1)$ cut each other at right angles.

OR

Determine the quadratic curve $y = f(x)$ if it touches the line $y = x$ at the point $x = 1$ and passes through the point $(-1, 0)$.

15. Let $A = Q \times Q$, where Q is the set of all rational numbers and $*$ be a binary operation on A defined by $(a, b) * (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. Then find

(i) the identity element of $*$ in A.

(ii) invertible elements of A, and hence write the inverse of elements $(5, 3)$ and $\left(\frac{1}{2}, 4\right)$.

OR

Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f : A \to B$ is a function defined by $f(x)=\frac{x-1}{x-2}$, show that f is one-one and onto. Hence, find f^{-1}.

16. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean and variance of the number of red cards.

17. Find the shortest distance between the lines whose vector equations are $\vec{r}=(b\hat{i}+2\hat{j}+2\hat{k})+\lambda(\hat{i}-2\hat{j}+2\hat{k})$ and $\vec{r}=(-4\hat{i}-\hat{k})+\mu(3\hat{i}-2\hat{j}-2\hat{k})$.

18. Differentiate the function $\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$ with respect to x.

OR

Find the value of k, for which the function

$$f(x)=\begin{cases}\dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1\le x<0\\[2ex] \dfrac{2x+1}{x-1}, & \text{if } 0\le x<1\end{cases}$$

is continuous at $x = 0$.

19. Using integration, find the area of the region enclosed between the circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

20. Water is leaking from a conical funnel at the rate of 5 cm^3/sec. If the radius of the base of the funnel is 5 cm and its altitude is 10 cm, find the rate at which the water level is dropping when it is 2.5 cm from the top.

21. If $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}, B=\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, then find the values of a and b.

22. Evaluate $\cos\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}\right)$.

23. Three bags contain balls as shown in the table below :

Bag	Number of white balls	Number of black balls	Number of red balls
I	1	2	3
II	2	1	1
III	4	3	2

A bag is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they came from the III bag?

SECTION - D

24. Prove that : $\int_0^{\pi/2} \frac{\sin^2 x}{(\sin x+\cos x)}dx=\frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$

OR

Prove that : $\int_0^{\pi/2} \frac{\sin^2 x}{(1+\sin x\cos x)}dx=\frac{\pi}{3\sqrt{3}}$

25. Find the vector equation of a plane passing through the intersection of the planes $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=6$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})=-5$ and the point $(1, 1, 1)$.

OR

Find the distance of the point $(-2, 3, -4)$ from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$, measured parallel to the plane $4x + 12y - 3z + 1 = 0$.

26. Find the differential equation of all the circles in the first quadrant which touch the coordinate axes.

OR

Solve the following initial value problems :

$y-x\frac{dy}{dx}=2\left(1+x^2\frac{dy}{dx}\right), y(1)=1.$

27. A company makes two types of belts, A and B; profits on these belts being ₹ 4 and ₹ 3 each respectively. Each belt of type A requires twice as much time as a belt of type B. Also, if all belts were of type B, the company could make 1000 belts per day. The supply of leather is sufficient for only 800 belts per day (both A and B combined). At the most 400 buckles for belts of type A and 700 for those of type B are available per day. How many belts of each type should the company make per day so as to maximize the profit?

28. Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius a is a square of side $\sqrt{2}a$.

29. Solve the system of the following equations:

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4;\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1;\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2.$$

SOLUTIONS

1. Let p = probability of getting an even number in a single throw.

$$\Rightarrow\ p=\frac{3}{6}=\frac{1}{2}\ \Rightarrow\ q=(1-p)=\left(1-\frac{1}{2}\right)=\frac{1}{2}$$

$$\therefore\ \text{Variance} = npq=\left(3\times\frac{1}{2}\times\frac{1}{2}\right)=\frac{3}{4}. \qquad [\because\ n=3]$$

2. The highest order differential coefficient in the given differential equation is $\frac{d^5y}{dx^5}$. So, order is 5. Since L.H.S. of the differential equation is not expressible as a polynomial in $\frac{dy}{dx}$. So, its degree is not defined.

OR

Order = 3, degree = 1

3. We have, $f(x) = 27x^3$ and $g(x) = x^{1/3}$

$\therefore\ gof(x) = g(f(x)) = g(27x^3) = (27x^3)^{1/3} = 3x.$

4. Given, $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}, \vec{b}=-\hat{i}+2\hat{j}+\hat{k}, \vec{c}=-3\hat{i}+\hat{j}+2\hat{k}$

$$\therefore\ [\vec{a}\,\vec{b}\,\vec{c}]=\begin{vmatrix}2&1&3\\-1&2&1\\-3&1&2\end{vmatrix}=6-1+15=20.$$

5. Let the direction cosines of the given line be l, m and n.

$\therefore\ l = \cos\alpha$, $m = \cos\beta$ and $n = \cos\gamma$.

Now, $l^2 + m^2 + n^2 = 1 \Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow\ 2\cos^2\alpha + 2\cos^2\beta + 2\cos^2\gamma = 2$

$\Rightarrow\ (1 + \cos 2\alpha) + (1 + \cos 2\beta) + (1 + \cos 2\gamma) = 2$

$\Rightarrow\ (\cos 2\alpha + \cos 2\beta + \cos 2\gamma) = -1$

6. Let $\tan^{-1}\sqrt{3}=\theta_1$. Then,

$$\tan\theta_1=\sqrt{3}=\tan\frac{\pi}{3}\ \Rightarrow\ \theta_1=\frac{\pi}{3}\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$$

Let $\sec^{-1}(-2) = \theta_2$. Then,

$$\sec\theta_2=-2=-\sec\frac{\pi}{3}=\sec\left(\pi-\frac{\pi}{3}\right)=\sec\frac{2\pi}{3}$$

$$\therefore\ \theta_2=\frac{2\pi}{3}\in[0,\pi]-\left\{\frac{\pi}{2}\right\}$$

$$\text{Hence, } \tan^{-1}\sqrt{3}-\sec^{-1}(-2)=\left(\frac{\pi}{3}-\frac{2\pi}{3}\right)=\frac{-\pi}{3}$$

7. Given, $2P(A)=P(B)=\frac{6}{13}$

$$\Rightarrow\ P(A)=\frac{3}{13}\ \text{and}\ P(B)=\frac{6}{13}$$

$$\text{Now, } P(A/B)=\frac{P(A\cap B)}{P(B)}$$

$$\Rightarrow\ P(A\cap B)=P(A/B)\cdot P(B)=\left(\frac{1}{3}\times\frac{6}{13}\right)=\frac{2}{13}.$$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$=\left(\frac{3}{13}+\frac{6}{13}-\frac{2}{13}\right)=\frac{(3+6-2)}{13}=\frac{7}{13}.$$

OR

$$P(\bar{A})=0.3\Rightarrow P(A)=0.7, P(B)=0.4\Rightarrow P(\bar{B})=0.6$$

$$P(A\cap\bar{B})=0.5\Rightarrow P(A\cap B)=P(A)-0.5=0.2$$

$$\text{Now, } P(B\,|\,A\cup\bar{B})=\frac{P[B\cap(A\cup\bar{B})]}{P(A\cup\bar{B})}$$

$$=\frac{P(B\cap A)}{P(A)+P(\bar{B})-P(A\cap\bar{B})}=\frac{0.2}{0.7+0.6-0.5}=\frac{0.2}{0.8}=\frac{1}{4}$$

8. We have, $(\vec{b}+\vec{c})=(\hat{i}+2\hat{j}-2\hat{k})+(2\hat{i}-\hat{j}+4\hat{k})$

$$=(1+2)\hat{i}+(2-1)\hat{j}+(-2+4)\hat{k}=(3\hat{i}+\hat{j}+2\hat{k})$$

$$\therefore\ \text{Projection of } (\vec{b}+\vec{c}) \text{ on } \vec{a}=\frac{(\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}|}$$

$$=\frac{(3\hat{i}+\hat{j}+2\hat{k})\cdot(2\hat{i}-2\hat{j}+\hat{k})}{\sqrt{2^2+(-2)^2+1^2}}=\frac{(6-2+2)}{\sqrt{9}}=\frac{6}{3}=2.$$

9. Here $f(x) = \log(x^2 + 2) - \log(3)$ is continuous in its domain and $f'(x)=\frac{2x}{x^2+2}$, which exists in $(-1, 1)$

Hence, f is derivable in $(-1, 1)$.

Also, $f(-1) = f(1) = 0$

Hence, all the conditions of Rolle's Theorem are satisfied. So, there exists $c \in (-1, 1)$ such that $f'(c) = 0$

$$\Rightarrow\ \frac{2c}{c^2+2}=0\ \Rightarrow\ c=0.$$

10. Dom (gof) = Dom $(f) = \{1, 3, 4\}$
$(gof)(1) = g\{f(1)\} = g(2) = 3$
$(gof)(3) = g\{f(3)\} = g(5) = 1$
$(gof)(4) = g\{f(4)\} = g(1) = 3$
$\therefore \quad gof = \{(1, 3), (3, 1), (4, 3)\}$

OR

We have, $* : R \times R \to R$ given by
$a * b = 2a + b$
$\therefore \quad (2 * 3) * 4 = (2 \times 2 + 3) * 4 = 2 \times 7 + 4 = 18.$

11. Given, $2A + B + X = O \quad \Rightarrow \quad X = -2A - B$

$$= -2\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ -6 & -8 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ -1 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 2-3 & -4+2 \\ -6-1 & -8-5 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}.$$

12. Let $I = \int \frac{(x-1)}{\sqrt{x+4}}\,dx$. Put $(x + 4) = t^2 \Rightarrow dx = 2t\,dt$

$$\therefore \quad I = 2\int \frac{(t^2-5)t}{t}dt = 2\int t^2 dt - 10\int dt = \frac{2t^3}{3} - 10t + C$$

$= \frac{2}{3}(x + 4)^{3/2} - 10(x + 4)^{1/2} + C.$

OR

$$\int_0^{\pi/2} x\cos x\,dx = [x\sin x]_0^{\pi/2} - \int_0^{\pi/2} 1\cdot\sin x\,dx$$

[Integrating by parts]

$$= \frac{\pi}{2} + [\cos x]_0^{\pi/2} = \frac{\pi}{2} - 1$$

13. We have, $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

$$\Rightarrow \quad \tan^{-1}\frac{\left\{\frac{x-1}{x-2} + \frac{x+1}{x+2}\right\}}{\left\{1 - \frac{(x-1)}{(x-2)}\cdot\frac{(x+1)}{(x+2)}\right\}} = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan^{-1}\left\{\frac{(x^2+x-2)+(x^2-x-2)}{-3}\right\} = \frac{\pi}{4}$$

$$\Rightarrow \quad \frac{2x^2-4}{-3} = \tan\frac{\pi}{4} = 1 \Rightarrow 2x^2 - 4 = -3$$

$$\Rightarrow \quad 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$$

14. Given curves are $x^2 = 9p\,(9 - y)$ ...(i)
and $x^2 = p\,(y + 1)$ ...(ii)
From (i) and (ii) we have,
$9p\,(9 - y) = p\,(y + 1) \Rightarrow 81 - 9y = y + 1$
$\Rightarrow \quad 10y = 80 \Rightarrow \quad y = 8$
Putting $y = 8$ in (i), we get $x^2 = 9p \Rightarrow x = \pm 3\sqrt{p}$
Thus, curves (i) and (ii) intersect at $P(3\sqrt{p}, 8)$ and $Q(-3\sqrt{p}, 8)$.

Differentiating (i) with respect to x, we get

$$2x = -9p\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = m_1 = -\frac{2x}{9p} = -\frac{2\times 3\sqrt{p}}{9p} = -\frac{2}{3\sqrt{p}}$$

Differentiating (ii) with respect to x, we get

$$2x = p\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = m_2 = \frac{2x}{p} \Rightarrow m_2 = \frac{2(3\sqrt{p})}{p} = \frac{6}{\sqrt{p}}$$

If curves (i) and (ii) cut each other at P at right angles, then $m_1 m_2 = -1$

$$\Rightarrow \quad -\frac{2}{3\sqrt{p}} \times \frac{6}{\sqrt{p}} = -1 \Rightarrow p = 4$$

Hence, the two curves cut each other at right angles, if $p = 4$.

OR

Let the required quadratic curve be $y = ax^2 + bx + c$ ...(i)
Since, it passes through $(-1, 0)$.
$\therefore \quad 0 = a - b + c$ ...(ii)
Differentiating (i) with respect to x, we get

$$\frac{dy}{dx} = 2ax + b \Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = 2a + b$$

Since the line $y = x$ touches curve (i) at $x = 1$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{x=1} = 1 \Rightarrow 2a + b = 1 \quad \text{...(iii)}$$

Putting $x = 1$ in $y = x$, we get $y = 1$. Thus, the curve (i) passes through $(1, 1)$.
$\therefore \quad 1 = a + b + c$ ...(iv)
Solving (ii), (iii) and (iv), we get
$a = 1/4$, $b = 1/2$ and $c = 1/4$

Substituting these values in (i), we get $y = \frac{x^2}{4} + \frac{x}{2} + \frac{1}{4}$, which is the required quadratic curve.

15. Given, $(a, b) * (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A = Q \times Q$.
(i) For identity element:
Let $(e_1, e_2) \in A$ is identity element.
$\therefore \quad (a, b) * (e_1, e_2) = (a, b) = (e_1, e_2) * (a, b)$
$\Rightarrow \quad (ae_1, b + ae_2) = (a, b) \quad \Rightarrow \quad ae_1 = a$ and $b + ae_2 = b$
$\Rightarrow \quad e_1 = 1$ and $e_2 = 0 \quad \therefore$ Identity element is $(1, 0)$
(ii) For invertible element:
Let $(a, b) \in A$ is invertible and $(x, y) \in A$ is inverse.
$\therefore \quad (a, b) * (x, y) = (1, 0)$

$\Rightarrow (ax, b+ay) = (1, 0) \Rightarrow ax = 1, b + ay = 0$

$\Rightarrow x = \frac{1}{a}, y = -\frac{b}{a}, a \neq 0$

$\therefore$ $(a, b) \in A$ is invertible if $a \neq 0$ and inverse is $\left(\frac{1}{a}, -\frac{b}{a}\right)$

Thus inverse of (5, 3) is $\left(\frac{1}{5}, -\frac{3}{5}\right)$ and inverse of $\left(\frac{1}{2}, 4\right)$ is (2, –8).

OR

Given $f(x) = \frac{x-1}{x-2}$

For one-one : Let $x_1, x_2 \in R - \{2\}$, then $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1-1}{x_1-2} = \frac{x_2-1}{x_2-2}$

$\Rightarrow x_1x_2 - x_2 - 2x_1 + 2 = x_1x_2 - x_1 - 2x_2 + 2 \Rightarrow x_1 = x_2$

As $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Hence, $f(x)$ is one-one.

For onto : Let for $y \in B = R - \{1\}$, then there exists $x \in A = R - \{2\}$, such that $y = f(x)$

$\therefore y = \frac{x-1}{x-2} \Rightarrow xy - x = 2y - 1 \Rightarrow x = \frac{2y-1}{y-1} \in A.$

Hence, $f(x)$ is onto.

Since $x = \frac{2y-1}{y-1} \Rightarrow f^{-1}(y) = \frac{2y-1}{y-1} \therefore f^{-1}(x) = \frac{2x-1}{x-1}.$

16. Let X denotes the number of red cards.

Now, X can take value 0, 1, 2

$$P(X=0) = \frac{{}^{26}C_2 \times {}^{26}C_0}{{}^{52}C_2} = \frac{26\times25}{52\times51} = \frac{25}{102}$$

$$P(X=1) = \frac{{}^{26}C_1 \times {}^{26}C_1}{{}^{52}C_2} = \frac{26\times26\times2}{52\times51} = \frac{26}{51} = \frac{52}{102}$$

$$P(X=2) = \frac{{}^{26}C_0 \times {}^{26}C_2}{{}^{52}C_2} = \frac{26\times25}{52\times51} = \frac{25}{102}$$

Hence, probability distribution is given by

X	$P(X)$	$XP(X)$	$X^2P(X)$
0	25/102	0	0
1	52/102	52/102	52/102
2	25/102	50/102	100/102
Total	1	102/102 = 1	152/102

Mean $(X) = \Sigma XP(X) = 1$

Variance $(X) = \Sigma X^2P(X) - (\text{Mean})^2 = 0.49$

17. The lines are $\vec{r} = (6\hat{i} + 2\hat{j} + 2\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$

and $\vec{r} = (-4\hat{i} - \hat{k}) + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$

$\vec{a}_1 = (6\hat{i} + 2\hat{j} + 2\hat{k}), \vec{b}_1 = (\hat{i} - 2\hat{j} + 2\hat{k}),$

$\vec{a}_2 = (-4\hat{i} - \hat{k})$ and $\vec{b}_2 = (3\hat{i} - 2\hat{j} - 2\hat{k})$

$\therefore (\vec{a}_2 - \vec{a}_1) = (-4\hat{i} - \hat{k}) - (6\hat{i} + 2\hat{j} + 2\hat{k}) = (-10\hat{i} - 2\hat{j} - 3\hat{k})$

and $(\vec{b}_1 \times \vec{b}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix} = 8\hat{i} + 8\hat{j} + 4\hat{k}$

$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{8^2 + 8^2 + 4^2} = 12$

$\therefore$ Shortest distance $= \left|\frac{(-10\hat{i} - 2\hat{j} - 3\hat{k}) \cdot (8\hat{i} + 8\hat{j} + 4\hat{k})}{12}\right|$

$= \left|\frac{-80 - 16 - 12}{12}\right| = \left|\frac{-108}{12}\right| = 9$ units.

18. Let $y = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$

Put $\frac{\pi}{4} + \frac{x}{2} = v$, then $\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) = \tan v = u$ (say)

$\therefore y = \log u, u = \tan v$ and $v = \frac{\pi}{4} + \frac{x}{2}$

$\Rightarrow \frac{dy}{du} = \frac{1}{u}, \frac{du}{dv} = \sec^2 v$ and $\frac{dv}{dx} = \frac{1}{2}$

Now, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dv} \times \frac{dv}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{u} \times \sec^2 v \times \frac{1}{2} = \frac{1}{\tan v} \sec^2 v \times \frac{1}{2}$ [$\because u = \tan v$]

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sin v \cos v} = \frac{1}{\sin\left(\frac{\pi}{2} + x\right)} = \frac{1}{\cos x} = \sec x.$

OR

Consider, $f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & -1 \le x < 0 \\ \frac{2x+1}{x-1}, & 0 \le x < 1 \end{cases}$

If $f(x)$ is continuous at $x = 0$, then

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$ ...(i)

Now, $f(0) = \frac{0+1}{0-1} = -1$ ...(ii)

$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} f(-h)$

$= \lim_{h \to 0} \frac{\sqrt{1+k(-h)} - \sqrt{1-k(-h)}}{-h}$

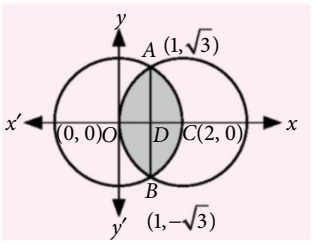
$$= \lim_{h\to 0} \frac{(\sqrt{1-kh}-\sqrt{1+kh})(\sqrt{1-kh}+\sqrt{1+kh})}{-h[\sqrt{1-kh}+\sqrt{1+kh}]}$$

$$= \lim_{h\to 0} \frac{-2kh}{-h[\sqrt{1-kh}+\sqrt{1+kh}]} = \frac{2k}{1+1} = k \quad \text{...(iii)}$$

$$\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(h) = \lim_{h\to 0} \frac{2(h)+1}{h-1} = \frac{0+1}{0-1} = -1 \quad \text{...(iv)}$$

From (i), (ii), (iii) and (iv), for $f(x)$ to be continuous at $x = 0$, we have $k = -1$

19. We have $x^2 + y^2 = 4$ ...(i)



a circle with centre at $O(0, 0)$ and radius = 2 units.
and $(x-2)^2 + y^2 = 4$ ...(ii)
a circle with its centre at $C(2, 0)$ and radius = 2 units.
Points of intersection of the two circles are $A(1, \sqrt{3})$ and $B(1, -\sqrt{3})$.

Required area = Area of shaded region

$$= 2\int_0^1 \sqrt{4-(x-2)^2}\,dx + 2\int_1^2 \sqrt{4-x^2}\,dx$$

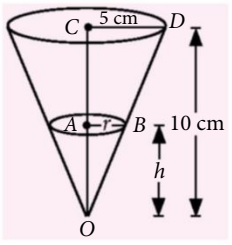
$$= 2\left[\frac{(x-2)\sqrt{4-(x-2)^2}}{2} + \frac{4}{2}\sin^{-1}\frac{(x-2)}{2}\right]_0^1$$

$$+2\left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_1^2$$

$$= 2\left\{\frac{-\sqrt{3}}{2} + 2\left(-\frac{\pi}{6}\right) - 2\left(-\frac{\pi}{2}\right) + \left(2\cdot\frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2\times\frac{\pi}{6}\right)\right\}$$

$$= 2\left(\frac{4\pi}{3} - \sqrt{3}\right) \text{ sq. units.}$$

20. Let r be the radius of the conical funnel, h the height of the conical funnel and V the volume of the water in the conical funnel at any instant t.



Then, $\frac{dV}{dt} = -5$ (given) ...(i)

From similar Δs OAB and OCD, we have

$$\frac{AB}{OA} = \frac{CD}{OC} \Rightarrow \frac{r}{h} = \frac{5}{10} = \frac{1}{2} \Rightarrow r = \frac{1}{2}h.$$

Now, $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\times\left(\frac{1}{2}h\right)^2 \times h = \frac{1}{12}\pi h^3$

$$\therefore \frac{dV}{dt} = \left(\frac{dV}{dh}\times\frac{dh}{dt}\right) = \frac{d}{dh}\left(\frac{1}{12}\pi h^3\right)\cdot\frac{dh}{dt} = \frac{\pi h^2}{4}\cdot\frac{dh}{dt}$$

$$\Rightarrow -5 = \frac{\pi h^2}{4}\cdot\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{-20}{\pi h^2}$$

$$\Rightarrow \left(\frac{dh}{dt}\right)_{h=7.5\text{cm}} = \frac{-20}{\pi\times(7.5)^2} = \frac{-16}{45\pi}\text{cm/sec.}$$

[$\because$ h = (10 – 2.5) cm = 7.5 cm]

Hence, the rate of change of water level at $h = 7.5$ cm is $\frac{-16}{45\pi}$ cm/sec.

21. Given, $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$

Now, $(A+B)^2 = (A+B)(A+B) = AA + AB + BA + BB$

$\Rightarrow$ $(A+B)^2 = A^2 + AB + BA + B^2$

$\Rightarrow$ $AB + BA = O$ ...(i) [$\because$ $(A+B)^2 = A^2 + B^2$]

$$AB = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} a-b & 2 \\ 2a-b & 3 \end{bmatrix}$$

$$BA = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} a+2 & -a-1 \\ b-2 & -b+1 \end{bmatrix}$$

From (i), $\begin{bmatrix} a-b & 2 \\ 2a-b & 3 \end{bmatrix} + \begin{bmatrix} a+2 & -a-1 \\ b-2 & -b+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2a-b+2 & 1-a \\ 2a-2 & 4-b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow$ $2a - b + 2 = 0$; $1 - a = 0$, $2a - 2 = 0$

$\Rightarrow$ $a = 1$ and $b = 4$

22. Let $\sin^{-1}\frac{3}{5} = A$ and $\sin^{-1}\frac{5}{13} = B$. Then,

$A, B \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ $\Rightarrow$ $\cos A > 0$ and $\cos B > 0$.

$\therefore$ $\sin A = \frac{3}{5}$ and $\sin B = \frac{5}{13}$

$$\Rightarrow \cos A = \sqrt{1-\sin^2 A} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\text{and } \cos B = \sqrt{1-\sin^2 B} = \sqrt{1-\frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$$

$$\therefore \cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13}\right) = \cos(A+B)$$

$$= \cos A\cos B - \sin A\sin B$$

$$= \left(\frac{4}{5}\times\frac{12}{13}\right) - \left(\frac{3}{5}\times\frac{5}{13}\right) = \frac{48}{65} - \frac{15}{65} = \frac{33}{65}.$$

23. Let A, B and C denotes the events of choosing bag I, II and III respectively and E denotes the event that one white and one red ball is drawn.

Now, $P(A) = P(B) = P(C) = 1/3$

Also, $P(E/A) = \dfrac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \dfrac{1\times 3\times 2}{6\times 5} = \dfrac{1}{5}$

$P(E/B) = 1/3$, $P(E/C) = 2/9$

$$\therefore \quad P(C/E) = \frac{P(C)P(E/C)}{P(A)P(E/A)+P(B)P(E/B)+P(C)P(E/C)}$$

$$= \frac{\frac{1}{3}\times\frac{2}{9}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{9}} = \frac{\frac{2}{9}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{9}} = \frac{2}{9}\times\frac{45}{34} = \frac{5}{17}$$

24. Let $I = \displaystyle\int_0^{\pi/2} \frac{\sin^2 x}{(\sin x+\cos x)}dx$ ...(i)

$$I = \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2}-x\right)}{\left[\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)\right]}dx = \int_0^{\pi/2} \frac{\cos^2 x}{(\sin x+\cos x)}dx \quad ...(ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{(\sin^2 x+\cos^2 x)}{(\sin x+\cos x)}dx = \int_0^{\pi/2} \frac{dx}{\sin x+\cos x}$$

$$= \int_0^{\pi/2} \frac{dx}{\left[\frac{2\tan(x/2)}{1+\tan^2(x/2)}+\frac{1-\tan^2(x/2)}{1+\tan^2(x/2)}\right]}$$

$$= \int_0^{\pi/2} \frac{\sec^2(x/2)}{[1-\tan^2(x/2)+2\tan(x/2)]}dx$$

$$= \int_0^1 \frac{2dt}{(1-t^2+2t)}, \text{where } \tan\frac{x}{2} = t$$

$$= 2\int_0^1 \frac{dt}{(\sqrt{2})^2-(t-1)^2} = 2\cdot\frac{1}{2\sqrt{2}}\left[\log\left|\frac{\sqrt{2}+(t-1)}{\sqrt{2}-(t-1)}\right|\right]_0^1$$

$$= \frac{1}{\sqrt{2}}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right) = \frac{1}{\sqrt{2}}\log\left[\frac{(\sqrt{2}+1)}{(\sqrt{2}-1)}\times\frac{(\sqrt{2}+1)}{(\sqrt{2}+1)}\right]$$

$$= \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)^2 = \sqrt{2}\log(\sqrt{2}+1)$$

$$\therefore \quad I = \frac{1}{2}\sqrt{2}\log(\sqrt{2}+1) = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1).$$

OR

Let $I = \displaystyle\int_0^{\pi/2} \frac{\sin^2 x}{(1+\sin x\cos x)}dx$ ...(i)

$$I = \int_0^{\pi/2} \frac{\sin^2[(\pi/2)-x]}{1+\sin[(\pi/2)-x]\cos[(\pi/2)-x]}dx = \int_0^{\pi/2} \frac{\cos^2 x}{(1+\sin x\cos x)}dx \quad ...(ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{(\sin^2 x+\cos^2 x)}{(1+\sin x\cos x)}dx = \int_0^{\pi/2} \frac{dx}{(1+\sin x\cos x)}$$

$$= \int_0^{\pi/2} \frac{\sec^2 x}{(\sec^2 x+\tan x)}dx$$

$$= \int_0^{\pi/2} \frac{\sec^2 x}{(1+\tan^2 x+\tan x)}dx = \int_0^{\infty} \frac{dt}{(t^2+t+1)}, \text{ where } \tan x = t$$

$$= \int_0^{\infty} \frac{dt}{\left(t+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \left[\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right)\right]_0^{\infty}$$

$$= \frac{2}{\sqrt{3}}\cdot\left(\frac{\pi}{2}-\frac{\pi}{6}\right) = \frac{2\pi}{3\sqrt{3}}$$

Hence, $I = \dfrac{\pi}{3\sqrt{3}}$

25. Here, $\vec{n}_1 = \hat{i}+\hat{j}+\hat{k}$ and $\vec{n}_2 = 2\hat{i}+3\hat{j}+4\hat{k}$

and $d_1 = 6$ and $d_2 = -5$

Hence, using the relation $\vec{r}\cdot(\vec{n}_1+\lambda\vec{n}_2) = d_1+\lambda d_2$, we get

$$\vec{r}\cdot[\hat{i}+\hat{j}+\hat{k}+\lambda(2\hat{i}+3\hat{j}+4\hat{k})] = 6-5\lambda$$

$$\Rightarrow \quad \vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}] = 6-5\lambda \quad ...(1)$$

Taking $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$, we get

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}] = 6-5\lambda$

$\Rightarrow$ $(1+2\lambda)x + (1+3\lambda)y + (1+4\lambda)z = 6-5\lambda$

$\Rightarrow$ $(x+y+z-6)+\lambda(2x+3y+4z+5) = 0$ ...(2)

Given that, the plane passes through the point (1, 1, 1).

So the point (1, 1, 1) must satisfy equation (2).

i.e., $(1+1+1-6)+\lambda(2+3+4+5) = 0 \Rightarrow \lambda = \dfrac{3}{14}$

Putting the value of λ in equation (1), we get

$$\vec{r}\cdot\left[\left(1+\frac{3}{7}\right)\hat{i}+\left(1+\frac{9}{14}\right)\hat{j}+\left(1+\frac{6}{7}\right)\hat{k}\right] = 6-\frac{15}{14}$$

or $\vec{r}\cdot\left(\dfrac{10}{7}\hat{i}+\dfrac{23}{14}\hat{j}+\dfrac{13}{7}\hat{k}\right) = \dfrac{69}{14}$

or $\vec{r}\cdot(20\hat{i}+23\hat{j}+26\hat{k}) = 69$

which is the required vector equation of the plane.

OR

The equation of the given plane is

$4x + 12y - 3z + 1 = 0$. ...(i)

Let $P(-2, 3, -4)$ be the given point.

Let l be given line whose equation is

$\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5} = \lambda$(say). ...(ii)

The general point on this line is $\left(3\lambda - 2, \frac{4\lambda-3}{2}, \frac{5\lambda-4}{3}\right)$.

For some value of λ, let the point $Q\left(3\lambda - 2, \frac{4\lambda-3}{2}, \frac{5\lambda-4}{3}\right)$ lie on the line (ii) such that PQ is parallel to the given plane (i).

D.r.'s of PQ are $3\lambda, \frac{4\lambda-9}{2}, \frac{5\lambda+8}{3}$.

D.r's of the normal to the given plane are 4, 12, –3.

Now, PQ is parallel to the given plane (i).

$\Rightarrow$ PQ is perpendicular to the normal to the plane (i).

$\Rightarrow (4 \times 3\lambda) + 12 \times \frac{(4\lambda-9)}{2} - 3 \times \frac{(5\lambda+8)}{3} = 0$

$\Rightarrow 12\lambda + (24\lambda - 54) - (5\lambda + 8) = 0 \Rightarrow \lambda = 2.$

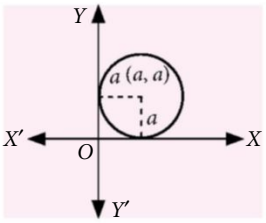
$\therefore$ Coordinates of Q are $\left(4, \frac{5}{2}, 2\right)$

$\Rightarrow PQ = \sqrt{(4+2)^2 + \left(\frac{5}{2} - 3\right)^2 + (2+4)^2} = 8.5$ units.

26. The equation of the family of circles in the first quadrant which touch the coordinate axes is

$(x - a)^2 + (y - a)^2 = a^2$, ...(i)

where a is a parameter.



Differentiating (i) with respect to x, we get

$2(x-a) + 2(y-a)\frac{dy}{dx} = 0$

$\Rightarrow x - a + (y-a)\frac{dy}{dx} = 0$

$\Rightarrow a = \frac{x + y\frac{dy}{dx}}{1 + \frac{dy}{dx}} \Rightarrow a = \frac{x+py}{1+p}$, where $p = \frac{dy}{dx}$

Substituting the value of a in (i), we get

$\left(x - \frac{x+py}{1+p}\right)^2 + \left(y - \frac{x+py}{1+p}\right)^2 = \left(\frac{x+py}{1+p}\right)^2$

$\Rightarrow (xp - py)^2 + (y - x)^2 = (x + py)^2$

$\Rightarrow (x - y)^2 (p^2 + 1) = (x + py)^2$

$\Rightarrow (x-y)^2\left\{1 + \left(\frac{dy}{dx}\right)^2\right\} = \left(x + y\frac{dy}{dx}\right)^2$, which is the required differential equation.

OR

We have, $y - x\frac{dy}{dx} = 2\left(1 + x^2\frac{dy}{dx}\right)$

$\Rightarrow y - 2 = x(2x+1)\frac{dy}{dx} \Rightarrow \int \frac{1}{x(2x+1)}dx = \int \frac{1}{y-2}dy$

$\Rightarrow \int\left(\frac{1}{x} - \frac{2}{(2x+1)}\right)dx = \int \frac{1}{y-2}dy$

$\Rightarrow \log|x| - \log|2x + 1| = \log|y - 2| + \log C$

$\Rightarrow \log\left|\frac{x}{2x+1}\right| - \log|y-2| = \log C$

$\Rightarrow \log\left|\frac{x}{2x+1} \times \frac{1}{y-2}\right| = \log C \Rightarrow \left|\frac{x}{(2x+1)(y-2)}\right| = C$ ...(i)

Putting $x = 1$ and $y = 1$ in (i), we get $C = 1/3$

Putting $C = \frac{1}{3}$ in (i), we get $\left|\frac{x}{(2x+1)(y-2)}\right| = \frac{1}{3}$

$\Rightarrow \frac{x}{(2x+1)(y-2)} = \pm\frac{1}{3}$

$\Rightarrow y - 2 = \pm\frac{3x}{2x+1} \Rightarrow y = 2 \pm \frac{3x}{2x+1}$

But, $y = 2 + \frac{3x}{2x+1}$ is not satisfied by $y(1) = 1$

Hence, $y = 2 - \frac{3x}{2x+1}$, where $x \neq -\frac{1}{2}$ is the required solution.

27. Let company makes x belts of type A and y belts of type B per day.

Now, 1000 belts of type B can be made in 1 day.

$\therefore$ 500 belts of type A can be made in 1 day.

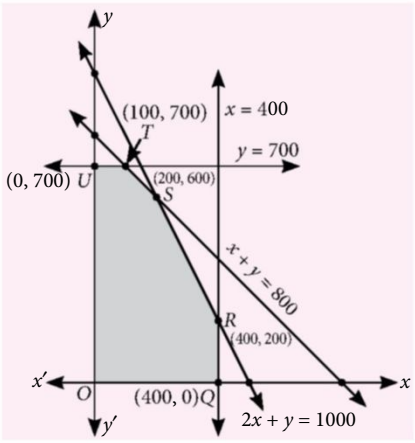
Thus, time taken to make x belts of type A and y belts of type $B = \left(\frac{x}{500} + \frac{y}{1000}\right)$ days.

$\therefore \frac{x}{500} + \frac{y}{1000} \leq 1$ *i.e.*, $2x + y \leq 1000$

Maximize, $Z = 4x + 3y$

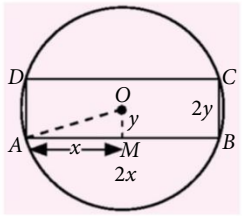
Subject to $x \geq 0$, $y \geq 0$, $x \leq 400$, $y \leq 700$, $x + y \leq 800$ and $2x + y \leq 1000$

The shaded region represents the feasible region, where $O(0, 0)$, $Q(400, 0)$, $R(400, 200)$, $S(200, 600)$, $T(100, 700)$, $U(0, 700)$.



Now, maximize $Z = 4x + 3y$
At $O(0, 0)$, $Z = 4(0) + 3(0) = 0$
At $Q(400, 0)$, $Z = 4(400) + 3(0) = 1600$
At $R(400, 200)$, $Z = 4(400) + 3(200) = 2200$
At $S(200, 600)$, $Z = 4(200) + 3(600) = 2600$
At $T(100, 700)$, $Z = 4(100) + 3(700) = 2500$
At $U(0, 700)$, $Z = 4(0) + 3(700) = 2100$
$\therefore$ Z is maximum when $x = 200$ and $y = 600$
Thus, the company should make 200 belts of type A and 600 belts of type B to have a maximum profit.

28. Let $ABCD$ be a rectangle in a given circle of radius a with centre at O. Let $AB = 2x$ and $AD = 2y$ be the sides of the rectangle. In ΔOAM



$AM^2 + OM^2 = OA^2$

$\Rightarrow \quad x^2 + y^2 = a^2$

$\Rightarrow \quad y = \sqrt{a^2 - x^2}$ ...(i)

Let P be the perimeter of the rectangle $ABCD$.

$\therefore \quad P = 4x + 4y \Rightarrow \quad P = 4x + 4\sqrt{a^2 - x^2}$

$\Rightarrow \quad \dfrac{dP}{dx} = 4 - \dfrac{4x}{\sqrt{a^2 - x^2}}$

The critical points of P are given by $\dfrac{dP}{dx} = 0$.

Put $\dfrac{dP}{dx} = 0 \Rightarrow 4 - \dfrac{4x}{\sqrt{a^2 - x^2}} = 0$

$\Rightarrow \sqrt{a^2 - x^2} = x \Rightarrow 2x^2 = a^2 \Rightarrow x = a/\sqrt{2}$

Now, $\dfrac{d^2P}{dx^2} = \dfrac{-4\left\{\sqrt{a^2 - x^2} - \dfrac{x(-x)}{\sqrt{a^2 - x^2}}\right\}}{\left\{\sqrt{a^2 - x^2}\right\}^2} = \dfrac{-4a^2}{(a^2 - x^2)^{3/2}}$

$\Rightarrow \quad \left(\dfrac{d^2P}{dx^2}\right)_{x=a/\sqrt{2}} = \dfrac{-4a^2}{\left(a^2 - \dfrac{a^2}{2}\right)^{3/2}} = \dfrac{-8\sqrt{2}}{a} < 0$

Thus, P is maximum when $x = a/\sqrt{2}$.

Putting $x = a/\sqrt{2}$ in (i), we get $y = a/\sqrt{2}$.

$\therefore \quad x = y = a/\sqrt{2} \Rightarrow 2x = 2y \Rightarrow AB = BC$

$\Rightarrow$ $ABCD$ is a square.

Hence, P is maximum when the rectangle is square of side $2x = \dfrac{2a}{\sqrt{2}} = \sqrt{2}\,a$.

29. We have, $\dfrac{2}{x} + \dfrac{3}{y} + \dfrac{10}{z} = 4$

$\dfrac{4}{x} - \dfrac{6}{y} + \dfrac{5}{z} = 1$; $\dfrac{6}{x} + \dfrac{9}{y} - \dfrac{20}{z} = 2$

which can be expressed in matrix forms as

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}\begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} \text{ or } AX = B$$

Solution is given by $X = A^{-1}B$ ...(i)

Now, $|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$

$= 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$
$= 150 + 330 + 720 = 1200 \neq 0$. Hence A^{-1} exists.

Cofactors of elements of A are

$A_{11} = 75, \quad A_{12} = 110, \quad A_{13} = 72$
$A_{21} = 150, \quad A_{22} = -100, \quad A_{23} = 0$
$A_{31} = 75, \quad A_{32} = 30, \quad A_{33} = -24$

$$\Rightarrow \text{adj}A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}' = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}\text{adj}\,A = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

From (i), $\begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \dfrac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$$\begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/5 \end{bmatrix}$$

$\Rightarrow \quad \dfrac{1}{x} = \dfrac{1}{2}, \dfrac{1}{y} = \dfrac{1}{3}, \dfrac{1}{z} = \dfrac{1}{5} \Rightarrow x = 2, y = 3, z = 5$

MPP MONTHLY Practice Problems

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. A variable plane makes with coordinate planes a tetrahedron of unit volume. The locus of the centroid of the tetrahedron is

(a) $xyz = 6$ (b) $xyz = \frac{3}{32}$

(c) $x + y + z = 6$ (d) $x^3 + y^3 + z^3 = 3$

2. Let $f'(x) < 0$ and $g'(x) > 0$ for all real x, then

(a) $f[g(x+1)] > f[g(x+5)]$ (b) $f[g(x)] < f[g(x+1)]$

(c) $g[f(x)] < g[f(x+2)]$ (d) Both (a) and (c)

3. Let a, b, c be non-zero real numbers such that $\int_0^1 (1+\cos^8 x)(ax^2+bx+c)dx = \int_0^2 (1+\cos^8 x)\times (ax^2+bx+c)dx$.

Then the quadratic equation $ax^2 + bx + c = 0$ has

(a) no root in (0, 2) (b) atleast one root in (1, 2)

(c) double root in (0, 2) (d) two imaginary roots

4. $\int\left(\frac{x}{x\sin x+\cos x}\right)^2 dx =$

(a) $\cot x + \frac{x\sec x}{x\sin x+\cos x} + c$

(b) $\tan x + \frac{x\sec x}{x\sin x+\cos x} + c$

(c) $\cot x - \frac{x\sec x}{x\sin x+\cos x} + c$

(d) $\tan x - \frac{x\sec x}{x\sin x+\cos x} + c$

5. If n is a natural number, then $\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}^n =$

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ if n is even

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ if n is odd

(c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ if n is a natural number

(d) none of these

6. The centroid of the triangle formed by (0, 0, 0) and the point of intersection of $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{1}$ with $x = 0$ and $y = 0$ is

(a) (1, 1, 1) (b) (1/6, –1/3, 1/6)

(c) (–1/6, 1/3, –1/6) (d) (1/3, 1/3, 1/3)

One or More Than One Option(s) Correct Type

7. The system of equations $-2x + y + z = a$; $x - 2y + z = b$; $x + y - 2z = c$ has

(a) no solution if $a + b + c \neq 0$

(b) unique solution if $a + b + c = 0$

(c) infinite number of solutions if $a + b + c = 0$

(d) none of these

8. The angle formed by the positive y-axis and the tangent to $y = x^2 + 4x - 17$ at $\left(\frac{5}{2}, \frac{-3}{4}\right)$ is

(a) $\tan^{-1}(9)$ (b) $\frac{\pi}{2} - \tan^{-1}(9)$

(c) $\frac{\pi}{2} + \tan^{-1}(9)$ (d) none of these

9. If $f(x) = \lim_{n\to\infty} (\sin x)^{2n}$, then f is

(a) continuous at $x = \pi$

(b) discontinuous at $x = \pi/2$

(c) discontinuous at $x = -\pi/2$

(d) discontinuous at an infinite number of points

10. If A and B are two independent events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{5}$, then

(a) $P\left(\frac{A}{B}\right) = \frac{1}{2}$ (b) $P\left(\frac{A}{A \cup B}\right) = \frac{5}{6}$

(c) $P\left(\frac{A}{A \cup B}\right) = \frac{6}{10}$ (d) none of these

11. The solution of $\frac{dy}{dx} = \frac{ax+h}{by+k}$ represents a parabola if

(a) $a = -2, b = 0$ (b) $a = -2, b = 2$

(c) $a = 0, b = 2$ (d) $a = 0, b = 0$

12. Let $f(x)$ be a non-constant twice differentiable function defined on $(-\infty, \infty)$ such that $f(x) = f(1-x)$ and $f'\left(\frac{1}{4}\right) = 0$, then

(a) $f'(x)$ vanishes at least twice on $[0, 1]$

(b) $f'\left(\frac{1}{2}\right) = 0$

(c) $\int_{-1/2}^{1/2} f\left(x + \frac{1}{2}\right) \sin x \, dx = 0$

(d) $\int_0^{1/2} f(t) e^{\sin \pi t} \, dt = \int_{1/2}^{1} f(1-t) e^{\sin \pi t} dt$

13. The vectors $a\hat{i} + 2a\hat{j} - 3a\hat{k}, (2a+1)\hat{i} + (2a+3)\hat{j} + (a+1)\hat{k}$ and $(3a+5)\hat{i} + (a+5)\hat{j} + (a+2)\hat{k}$ are non-coplanar for a in

(a) $\{0\}$ (b) $[0, \infty)$

(c) $(-\infty, 1)$ (d) $(1, \infty)$

Comprehension Type

Let $f(x) = \sin^{-1}(\sin x) + \cos^{-1}(\cos x)$, $0 \le x \le 2\pi$ and $g(x) = x^2$, $0 \le x \le 2\pi$

14. $f(x)$ increases in

(a) $(0, \pi)$ (b) $(\pi, 2\pi)$ (c) $\left(0, \frac{\pi}{2}\right)$ (d) $\left(\pi, \frac{3\pi}{2}\right)$

15. $f(g(x))$ decreases in

(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left(\frac{\pi}{2}, \pi\right)$

(c) $\left(\pi, \frac{3\pi}{2}\right)$ (d) $\left(\frac{3\pi}{2}, 2\pi\right)$

Matrix Match Type

16. Match the following.

Column-I		Column-II	
P.	$\lim_{x \to 0} \frac{\int_0^{x^2} \sec^2 t \, dt}{x \sin x} =$	1.	1
Q.	$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\sec x - \cos x} \, dx =$	2.	8
R.	Let $\frac{d}{dx} F(x) = \frac{e^{\sin x}}{x}$, $x > 0$ If $\int_1^2 \frac{2}{x} e^{\sin x^2} \cdot dx = F(k) - F(1)$, then $2k =$	3.	3
S.	The function y defined by the equation $xy - \log y = 1$ satisfies $x(yy'' + y'^2) - y'' + kyy' = 0$. The value of k is	4.	4

	P	Q	R	S
(a)	3	1	2	4
(b)	1	3	4	2
(c)	1	4	2	3
(d)	2	3	1	4

Numerical Answer Type

17. The value of $3a^2$ if the curves $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$ and $y^3 = 16x$ intersect orthogonally is

18. If $\vec{a} = (0, 1, -1)$ and $\vec{c} = (1, 1, 1)$ are given vectors, then $|\vec{b}|^2$, where $\vec{b}$ satisfies $\vec{a} \times \vec{b} + \vec{c} = 0$ and $\vec{a} \cdot \vec{b} = 3$ is

19. The area bounded by the curves $x = y^2$ and $x = 3 - 2y^2$ is

20. A fair coin is tossed n times. If the probability that head occurs six times is equal to the probability that head occurs eight times, then the value of n is

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SELF CHECK

No. of questions attempted .....
No. of questions correct .....
Marks scored in percentage .....

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

MPP MONTHLY Practice Problems

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. The mirror image of the directrix of the parabola $y^2 = 4(x + 1)$ in the line mirror $x + 2y = 3$ is

(a) $x = -2$ (b) $4y - 3x = 16$

(c) $x - 3y = 0$ (d) $x + y = 0$

2. If in a ΔABC, $\cos A + 2\cos B + \cos C = 2$, then a, b, c are in

(a) A.P. (b) G.P.

(c) H.P. (d) none of these

3. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is

(a) $\frac{17}{3^5}$ (b) $\frac{13}{3^5}$ (c) $\frac{11}{3^5}$ (d) $\frac{10}{3^5}$

4. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$

(a) is equal to $\frac{5}{2}$

(b) lies in the interval (1, 2)

(c) is strictly greater than $\frac{5}{2}$

(d) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

5. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q and r are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is

(a) $\frac{\sqrt{61}}{9}$ (b) $\frac{2\sqrt{17}}{9}$ (c) $\frac{\sqrt{34}}{9}$ (d) $\frac{2\sqrt{13}}{9}$

6. The eccentric angles of the extremities of latus-rectum to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are given by

(a) $\tan^{-1}\left(\pm\frac{ae}{b}\right)$ (b) $\tan^{-1}\left(\pm\frac{be}{a}\right)$

(c) $\tan^{-1}\left(\pm\frac{b}{ae}\right)$ (d) $\tan^{-1}\left(\pm\frac{a}{be}\right)$

One or More Than One Option(s) Correct Type

7. If $C_0, C_1, C_2, \ldots, C_n$ are coefficients in the binomial expansion of $(1 + x)^n$, then $C_0C_2 + C_1C_3 + C_2C_4 + \ldots + C_{n-2}C_n$ is equal to

(a) $\frac{2n!}{(n-2)!(n+2)!}$ (b) $\frac{2n!}{\{(n-2)!\}^2}$

(c) $\frac{2n!}{\{(n+2)!\}^2}$ (d) ${}^{2n}C_{n-2}$

EXAM ALERT 2019

Exam	Date
JEE Main II	7th to 20th April
VITEEE	10th to 21st April
SRMJEEE	15th to 25th April
UPSEE	21st April
KEAM	22nd & 23rd April
Karnataka CET	23rd & 24th April
AMU (Engg.)	28th April
COMEDK (Engg.)	12th May
BITSAT	16th to 26th May
JEE Advanced	19th May
WB JEE	26th May

8. If $e^x + e^{f(x)} = e$, then for $f(x)$

(a) domain = $(-\infty, 1)$ (b) range = $(-\infty, 1)$

(c) domain = $(-\infty, 0]$ (d) range = $(-\infty, 1]$

9. If $1, \omega, \omega^2, ..., \omega^{n-1}$ are the n, n^{th} roots of unity, then $(2-\omega)(2-\omega^2) .... (2-\omega^{n-1})$ equals

(a) $2^n - 1$

(b) ${}^nC_1 + {}^nC_2 + ... + {}^nC_n$

(c) $[{}^{2n}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n]^{1/2} - 1$

(d) $2^n + 1$

10. The p^{th} term T_p of H.P. is $q(p+q)$ and q^{th} term T_q is $p(p+q)$ when $p > 1, q > 1$, then

(a) $T_{p+q} = pq$ (b) $T_{pq} = p + q$

(c) $T_{p+q} > T_{pq}$ (d) $T_{pq} > T_{p+q}$

11. In triangle ABC, $\angle C = \frac{2\pi}{3}$, then the value of $\cos^2 A + \cos^2 B - \cos A \cdot \cos B$ is equal to

(a) $\frac{3}{4}$ (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

12. If n is a positive integer and $(3\sqrt{3}+5)^{2n+1} = \alpha + \beta$, where α is an integer and $0 < \beta < 1$, then

(a) α is an even integer

(b) $(\alpha + \beta)^2$ is divisible by 2^{2n+1}

(c) the integer just below $(3\sqrt{3}+5)^{2n+1}$ is divisible by 3

(d) α is divisible by 10

13. Which of the following is/are correct

(a) $101^{50} - 99^{50} > 100^{50}$

(b) $101^{50} - 100^{50} > 99^{50}$

(c) $(1000)^{1000} > (1001)^{999}$

(d) $(1001)^{999} > (1000)^{1000}$

Comprehension Type

Let p be a prime number and n be a positive integer, then exponent of p in $n!$ is denoted by $E_p(n!)$ and is given by

$$E_p(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + ... + \left[\frac{n}{p^k}\right]$$

where $p^k \le n < p^{k+1}$ and $[x]$ denotes the integral part of x.

If we isolate the power of each prime contained in any number N, then N can be written as $N = 2^{\alpha_1} \cdot 3^{\alpha_2} \cdot 5^{\alpha_3} \cdot 7^{\alpha_4} ....$, where α_i are whole numbers.

14. The last non zero digit in 20 ! must be equal to

(a) 2 (b) 4 (c) 6 (d) 8

15. The number of prime numbers among the numbers $105! + 2, 105! + 3, 105! + 4, ..., 105! + 104$ and $105! + 105$ is

(a) 31 (b) 32

(b) 33 (d) none of these

Matrix Match Type

16. A root of the equation on L.H.S. satisfies R.H.S :

	Column-I		Column-II
P.	$7\cos^2 x + \sin x \cos x - 3 = 0$	1.	$\tan x = \frac{4}{3}$
Q.	$2\sin^2\left(\frac{\pi}{2}\cos^2 x\right) = 1 - \cos(\pi \sin 2x)$	2.	$\tan x = -1$
R.	$6\sec^2 x - 11\tan x - 2 = 0$	3.	$\tan x = \frac{1}{2}$
S.	$2\cos 2x - \sin 2x = 2\sin^2 x$	4.	$\cos 2x = \frac{3}{5}$

	P	Q	R	S
(a)	1, 3	2, 4	1, 3, 4	3, 4
(b)	1, 2	3, 4	1, 3, 4	2, 3, 4
(c)	2, 4	3, 4	1, 3, 4	2, 4
(d)	3, 4	1, 2	1, 2, 4	2, 4

Numerical Answer Type

17. If $(10)^9 + 2(11)^1 (10)^8 + 3(11)^2 (10)^7 + ... + 10(11)^9 = k(10)^9$, then k is equal to

18. The value of $786\sqrt{e} \lim_{x \to 0} \left\{\frac{(1+x)^{1/x}}{e}\right\}^{1/x}$ must be

19. The least value of $\text{cosec}^2 x + 25\sec^2 x$ is

20. If $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ and the side $a = 2$, then area of triangle is k. The value of $2k^2$ is

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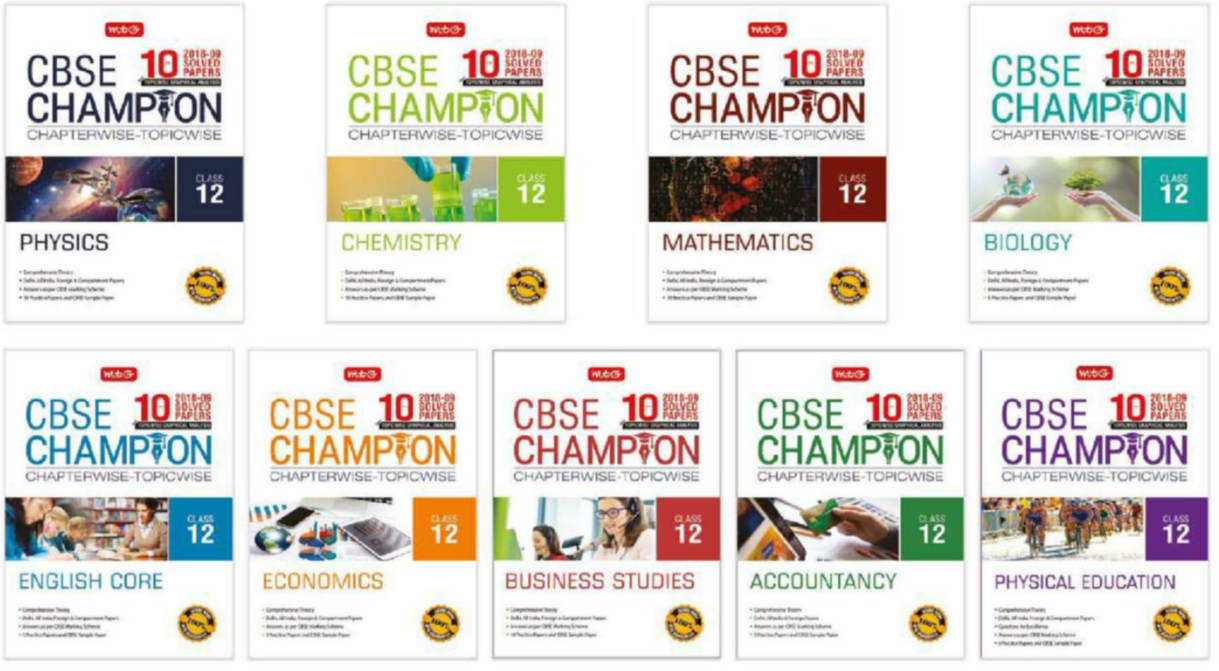

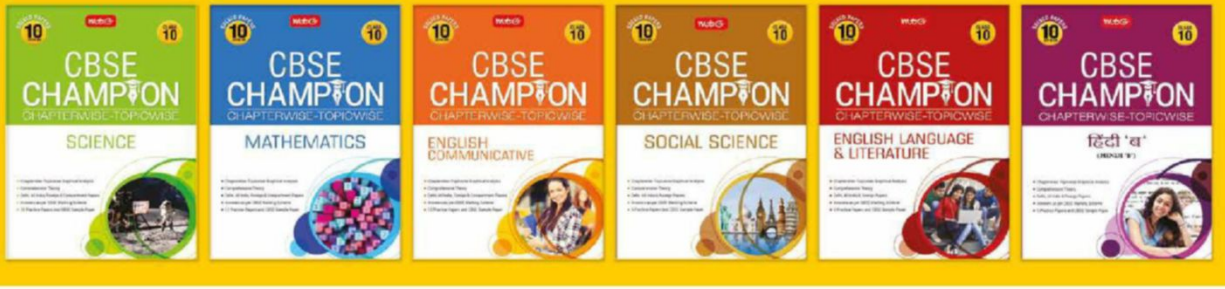

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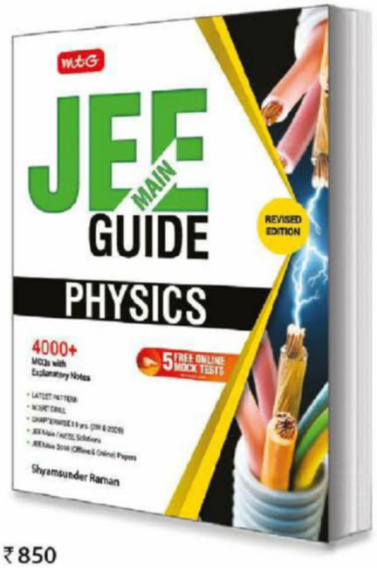

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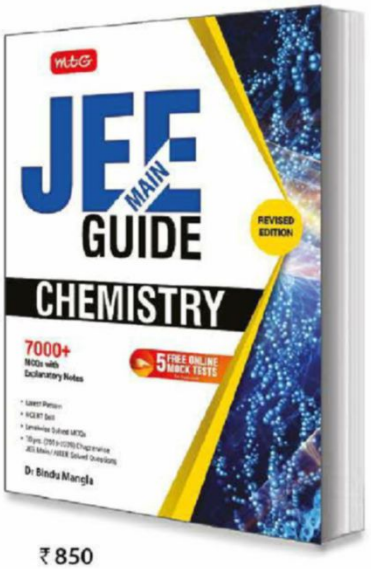

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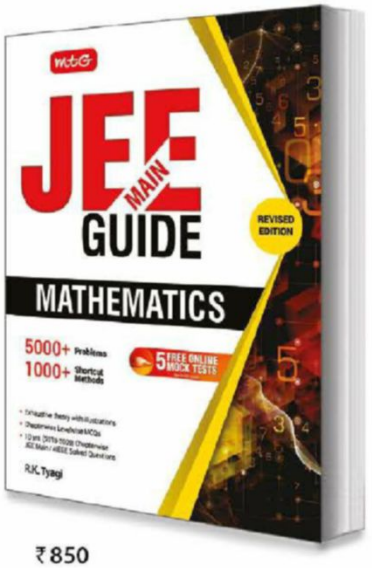

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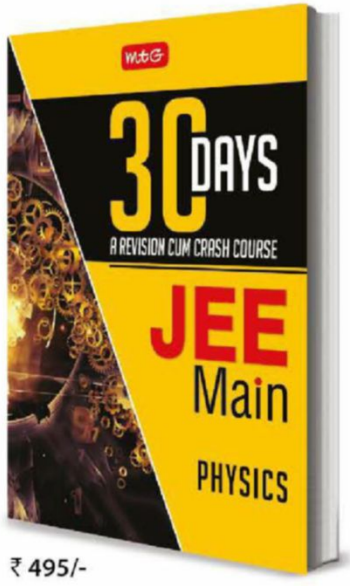

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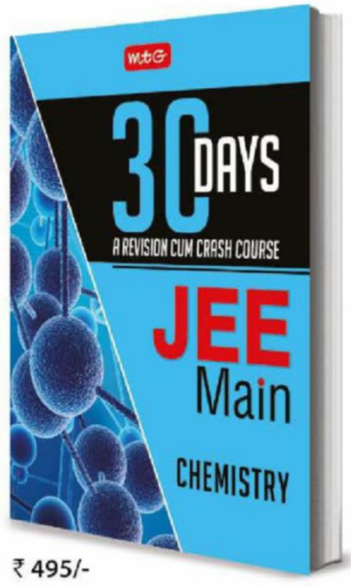

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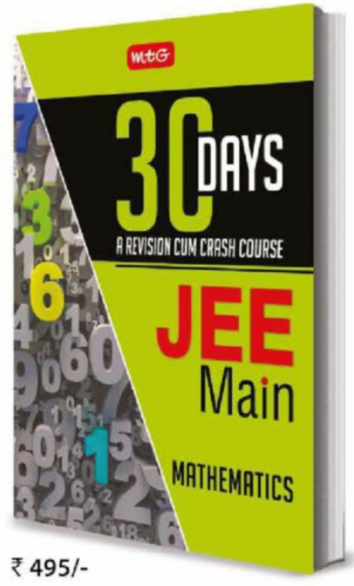

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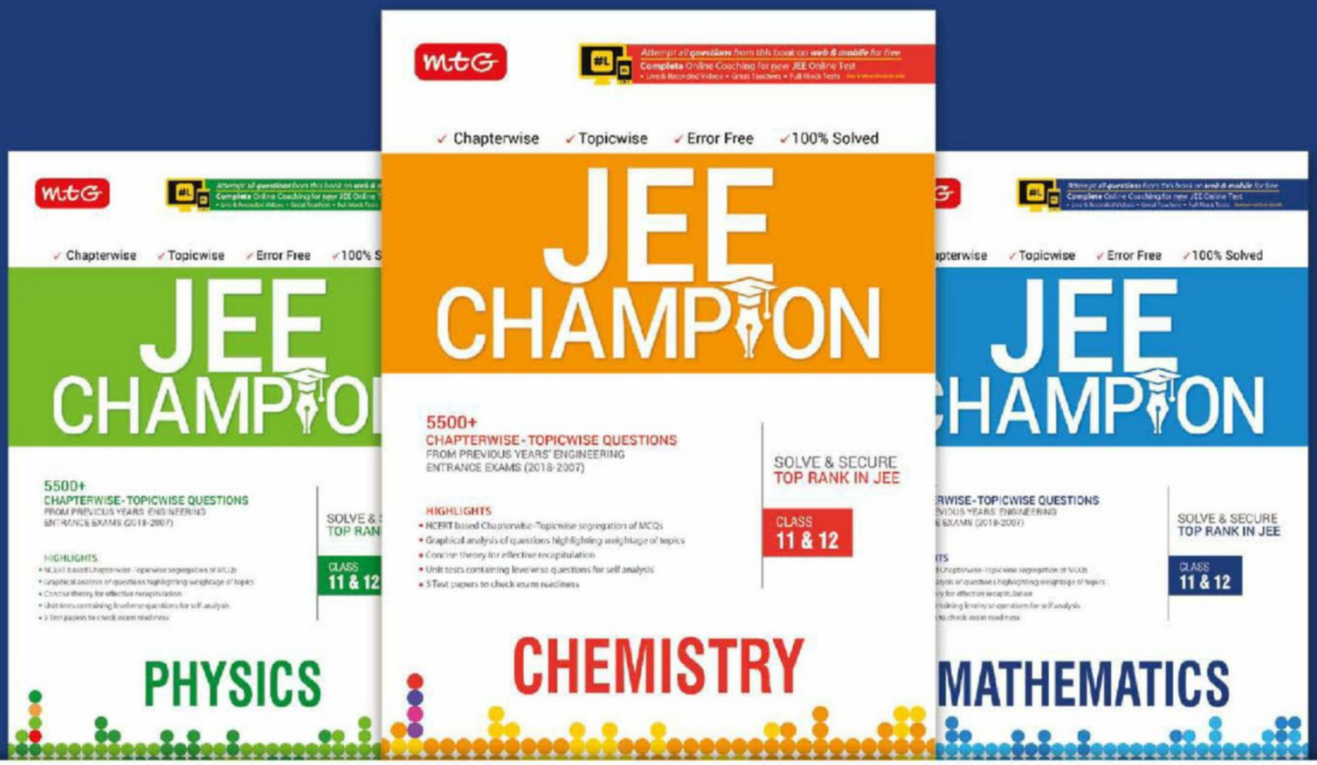

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